Photonic Crystals:

Periodic Surprises in Electromagnetism

Steven G. Johnson

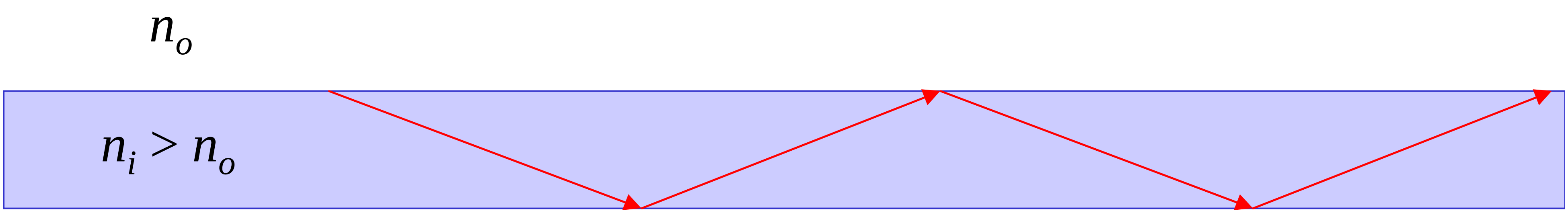
MIT

Complete Band Gaps:

You can leave home without them.

How else can we confine light?

Total Internal Reflection



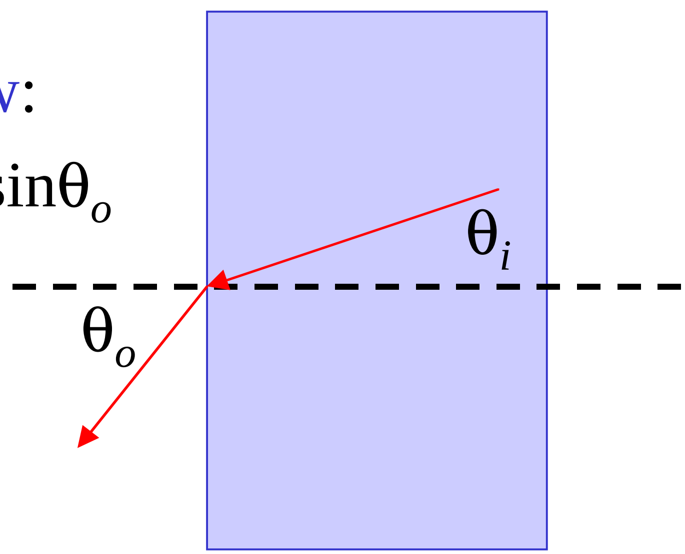
rays at shallow angles $> \theta_c$
are totally reflected

$\sin\theta_c = n_o / n_i$

< 1, so θ_c is real

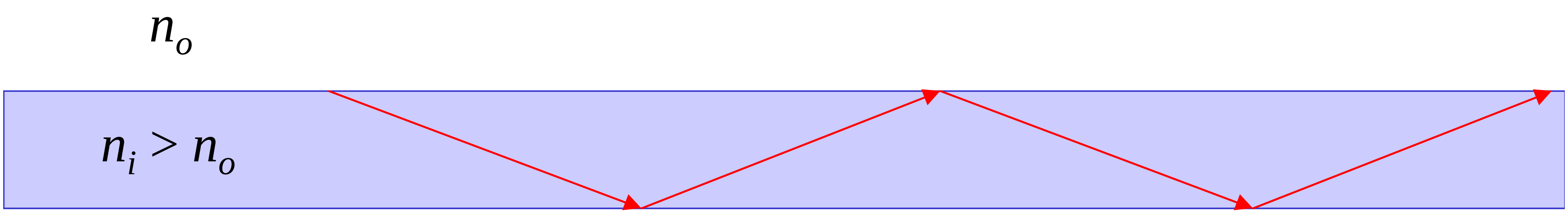
Snell's Law:

$n_i \sin\theta_i = n_o \sin\theta_o$



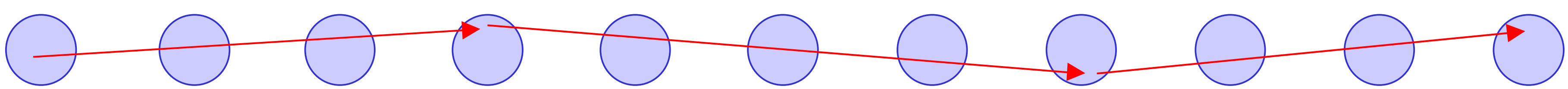
i.e. TIR can only guide
within higher index
unlike a band gap

Total Internal Reflection?



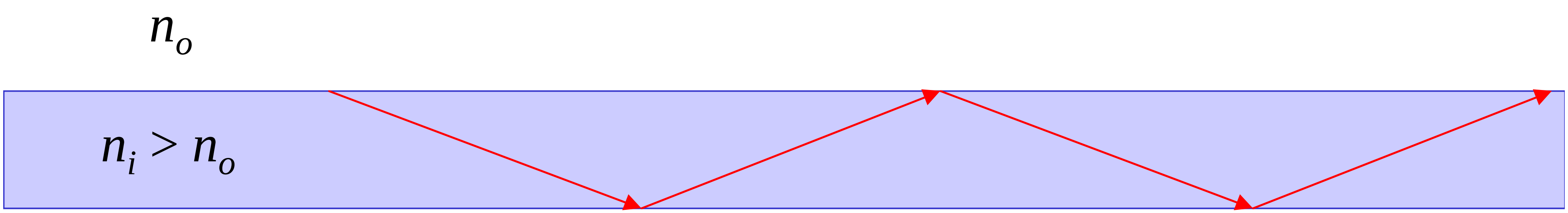
rays at shallow angles > θ_c
are totally reflected

So, for example,
a discontiguous structure can't possibly guide by TIR…



the rays can't stay inside!

Total Internal Reflection?



rays at shallow angles > θ_c
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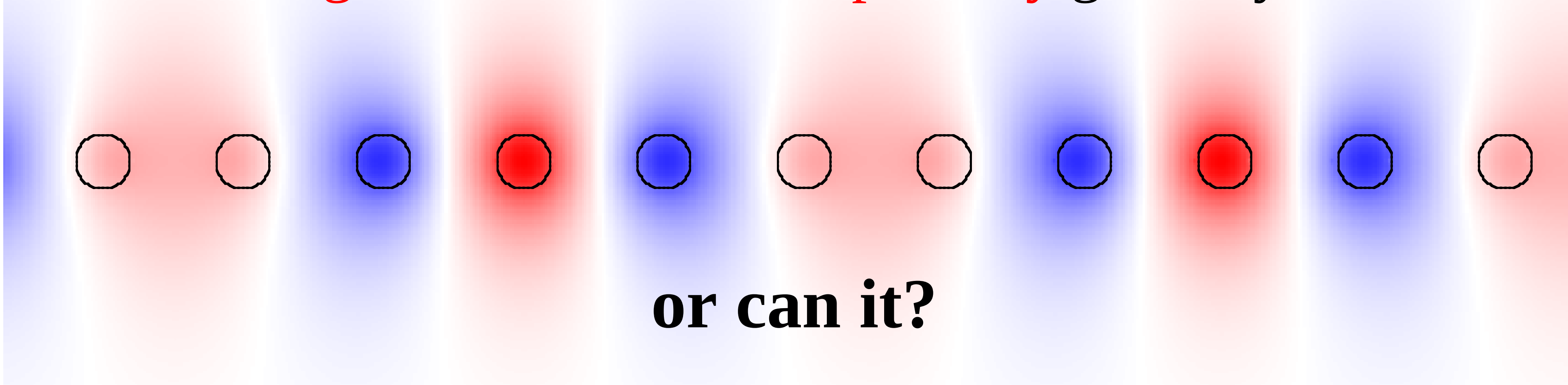
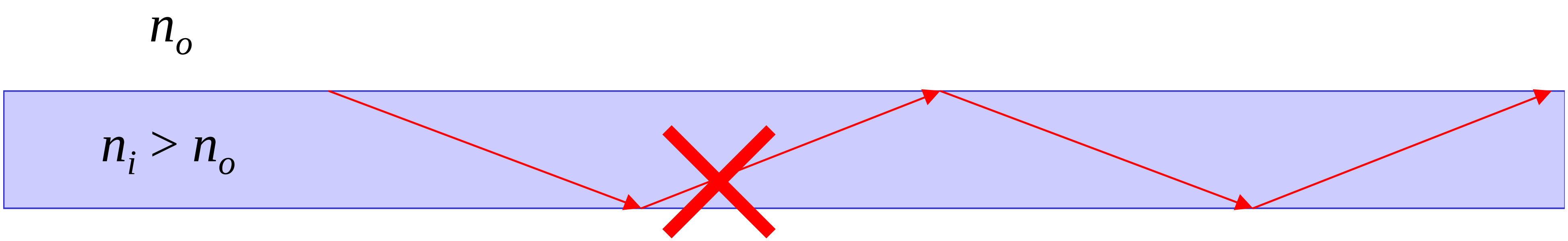
So, for example,
a discontiguous structure can't possibly guide by TIR…



or can it?

Total Internal Reflection Redux



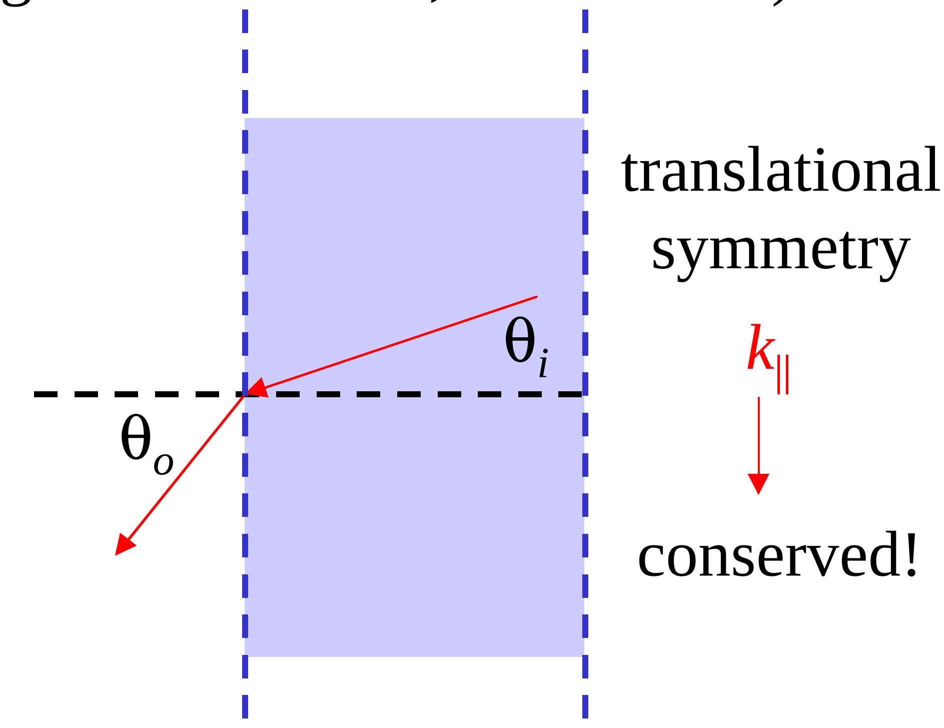
ray-optics picture is invalid on λ scale
(neglects coherence, near field…)

Snell's Law is really
conservation of $k_{\parallel}$ and ω:

$$|k_i| \sin\theta_i = |k_o| \sin\theta_o$$

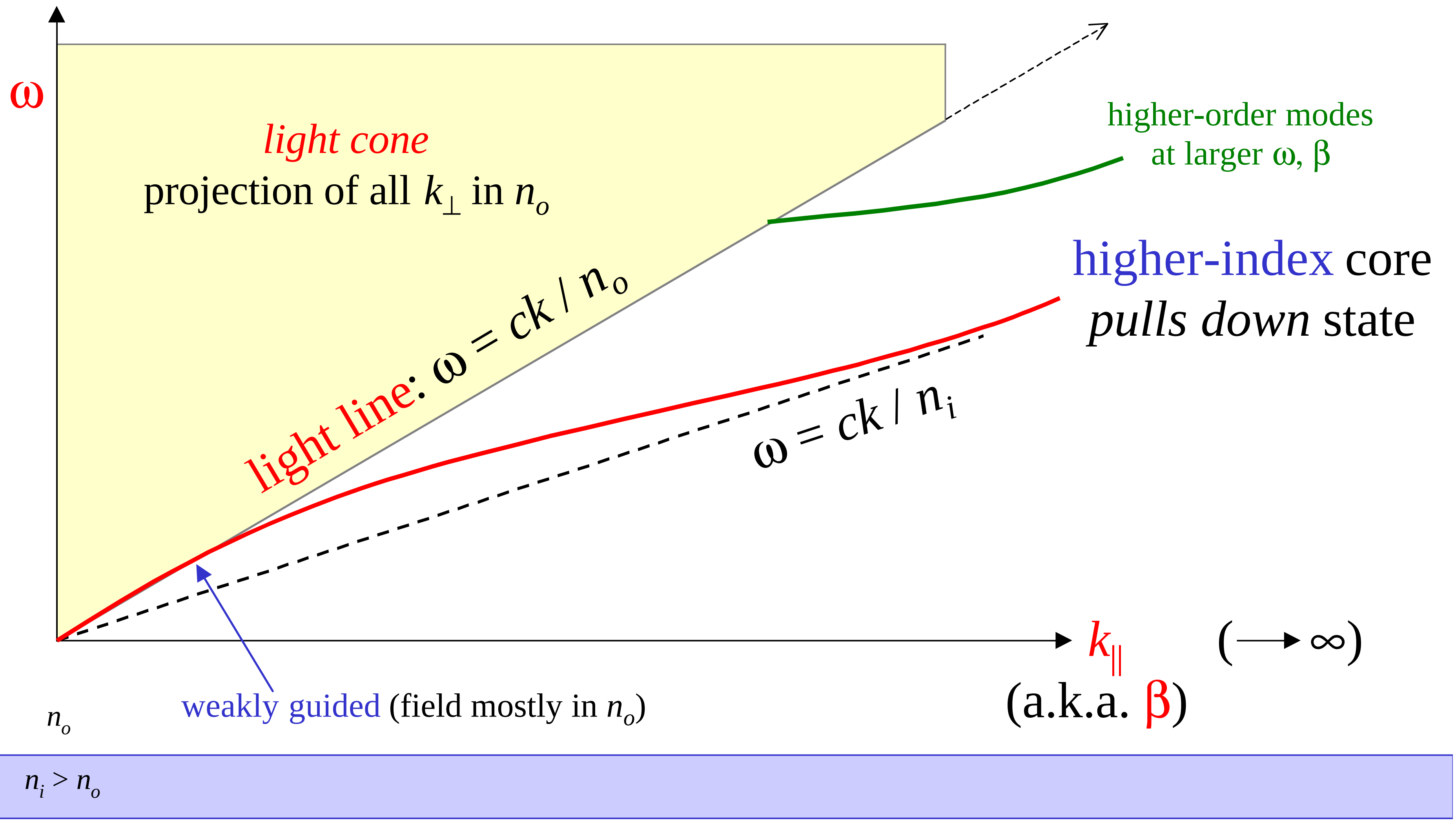
$$|k| = n\omega/c$$

(wavevector) (frequency)

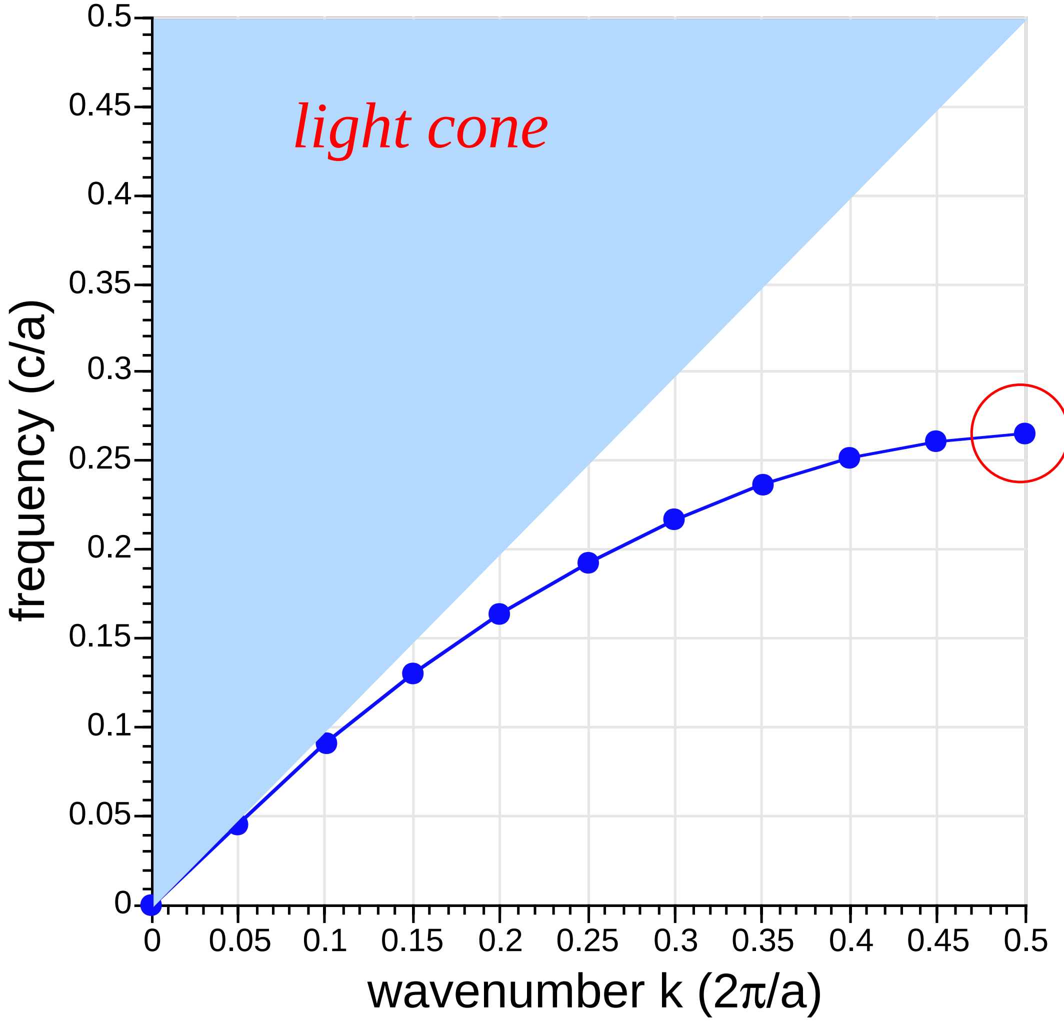


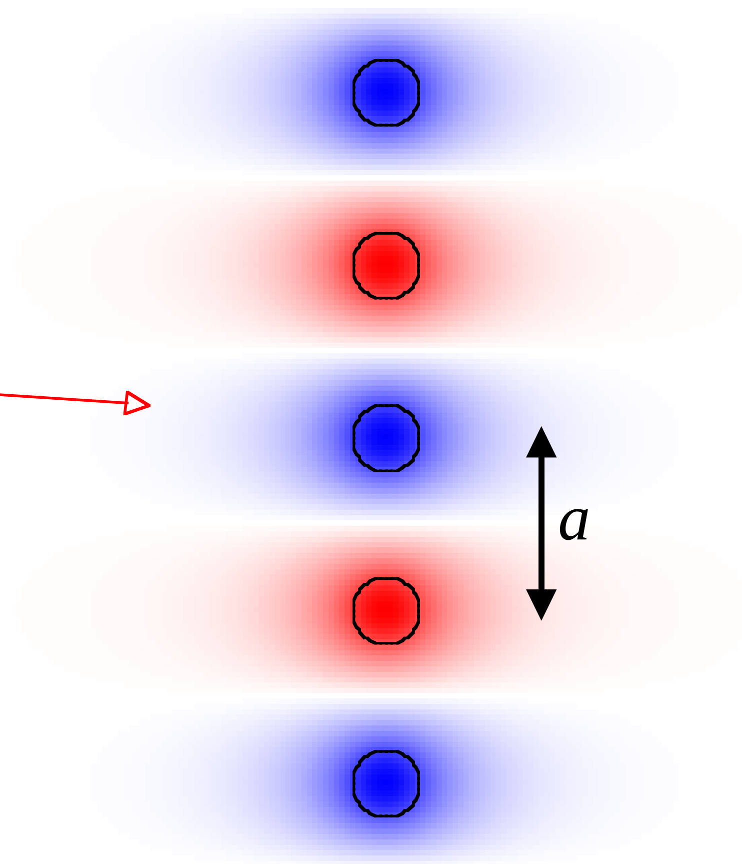
Waveguide Dispersion Relations

i.e. projected band diagrams



Strange ~~Total Internal Reflection~~ *Index Guiding*

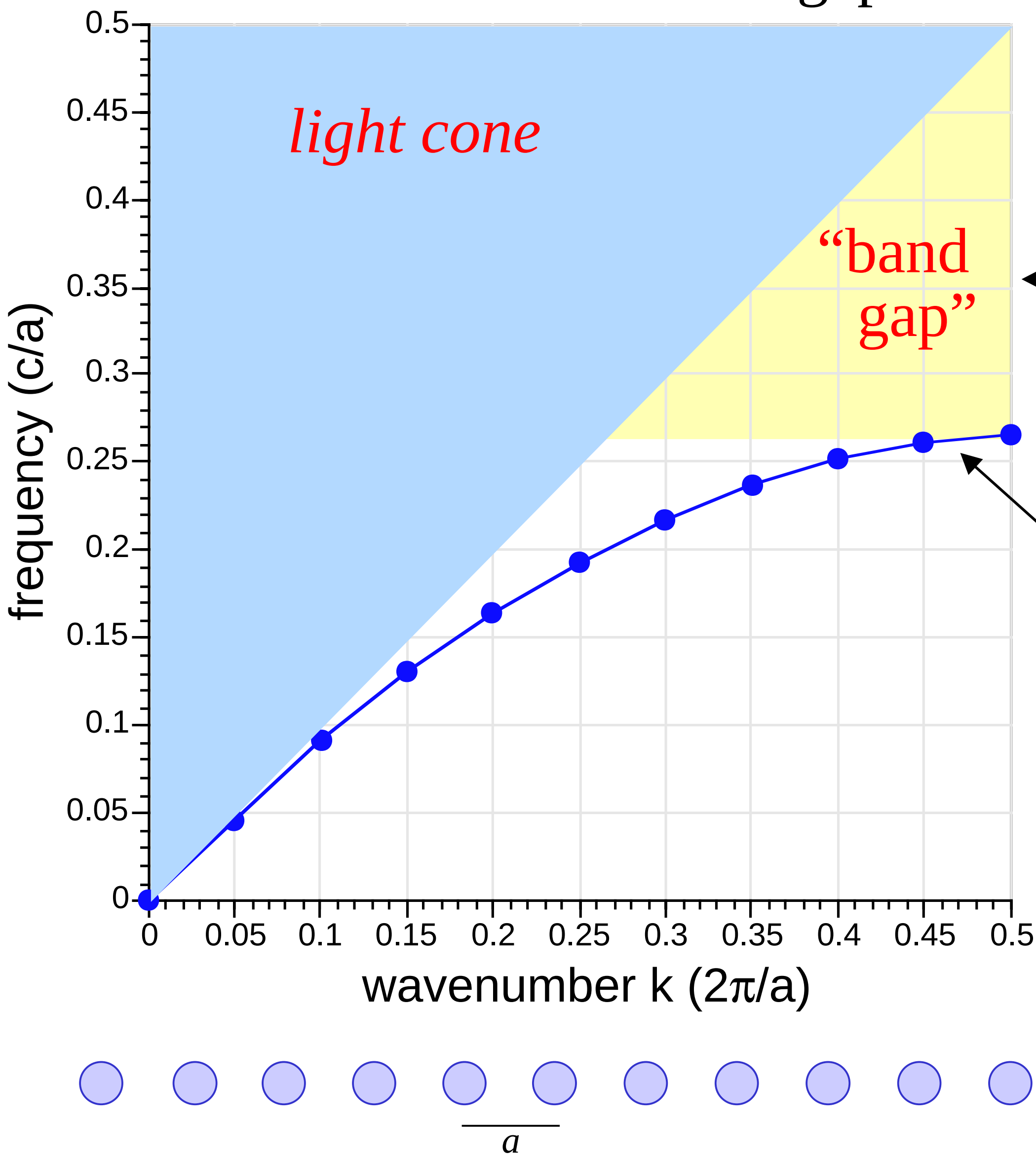




Conserved k and ω
+ higher index to pull down state
= localized/guided mode.

A Hybrid Photonic Crystal:

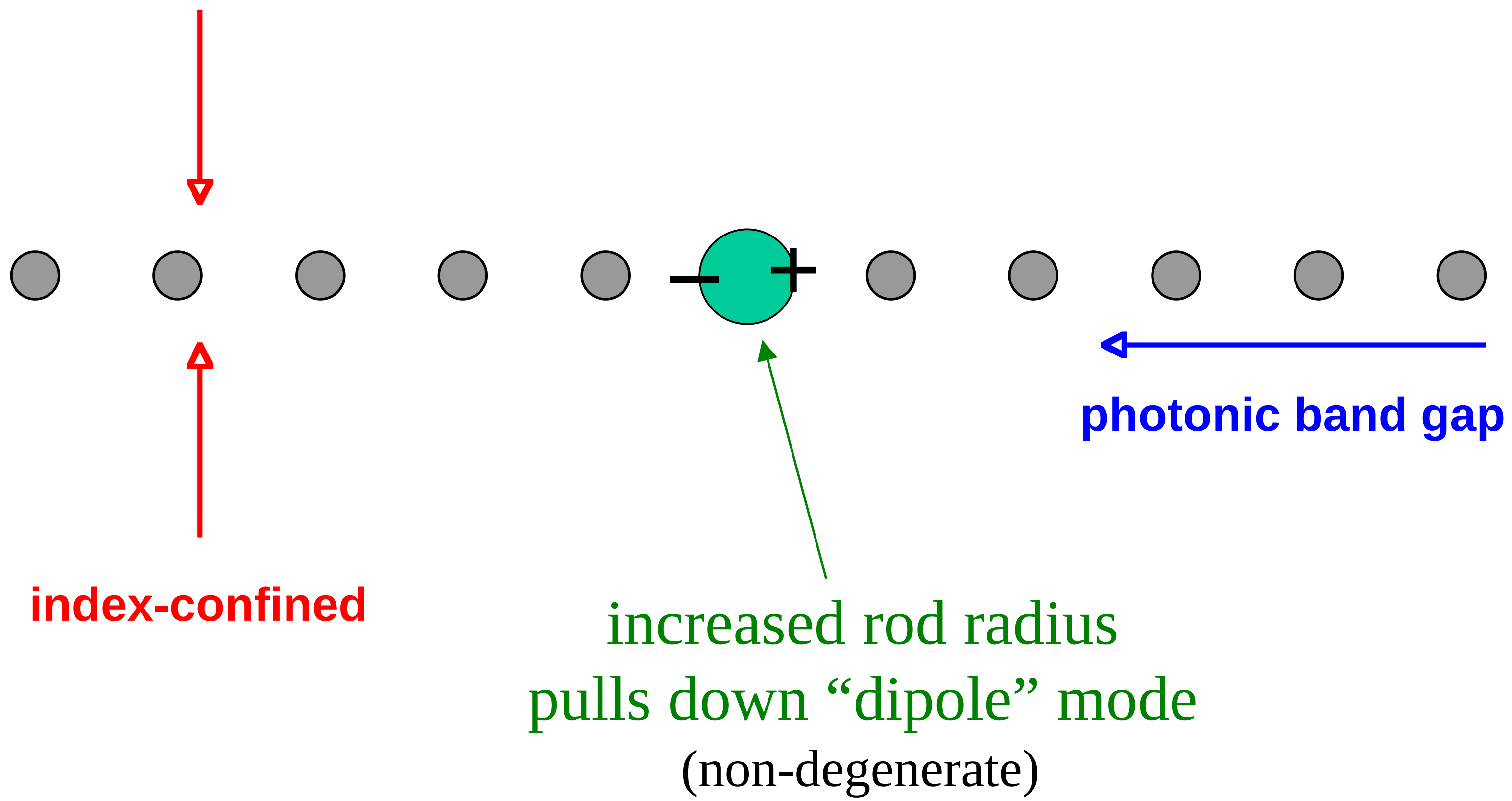
1d band gap + index guiding



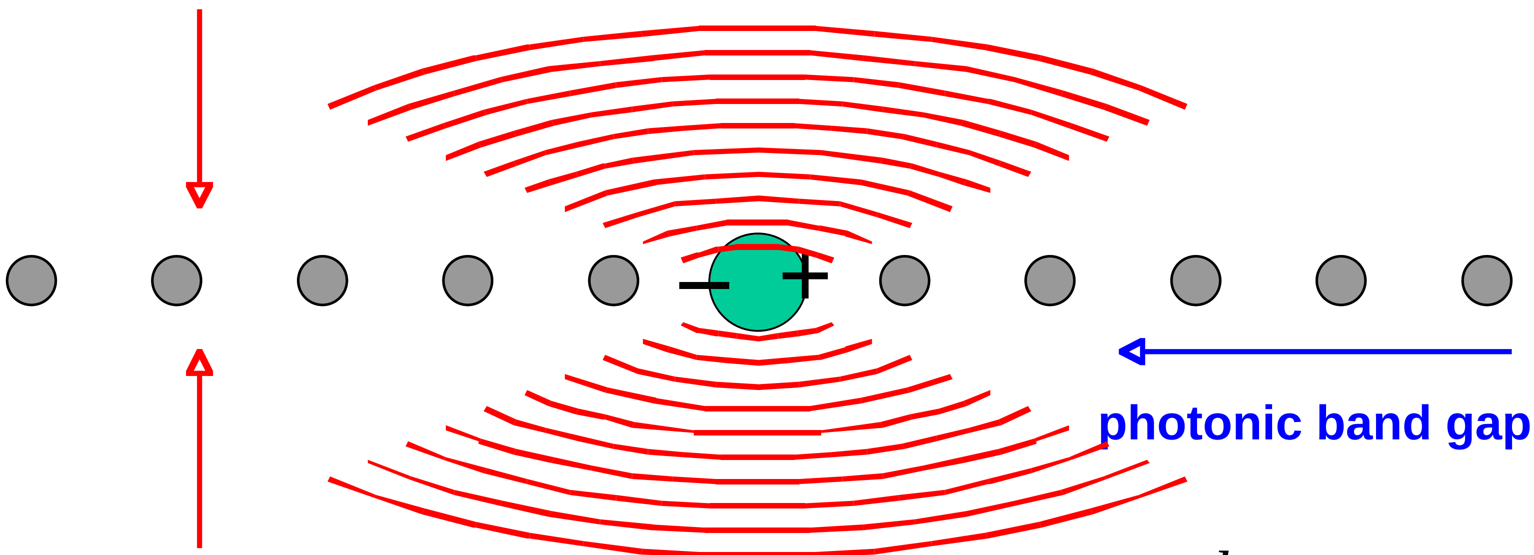
range of frequencies in which there are no guided modes

slow-light band edge

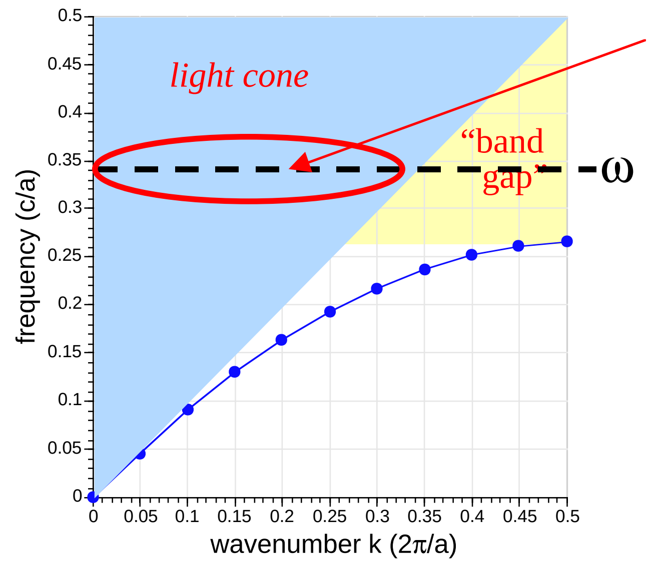
A Resonant Cavity



A Resonant Cavity



index-confined

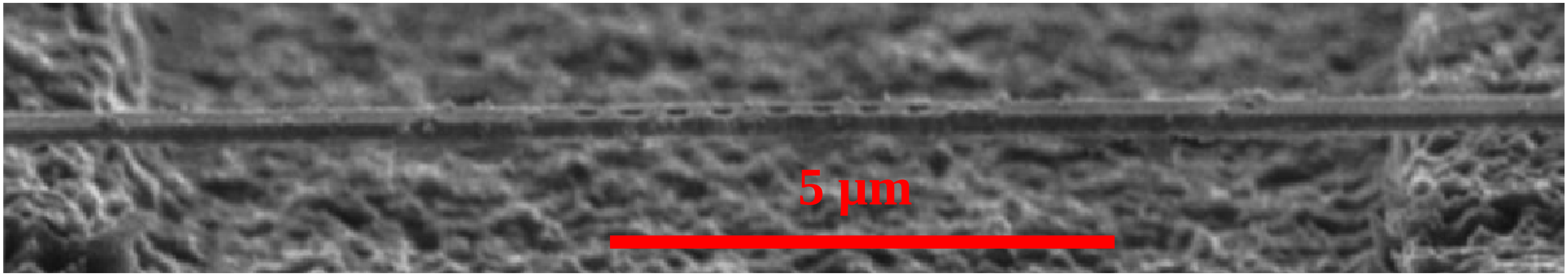


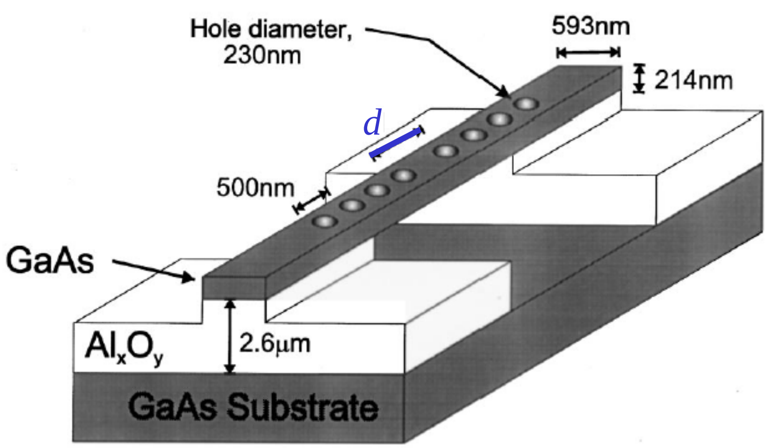
k not conserved
so coupling to
light cone:
radiation

The trick is to
keep the
radiation small…
(more on this later)

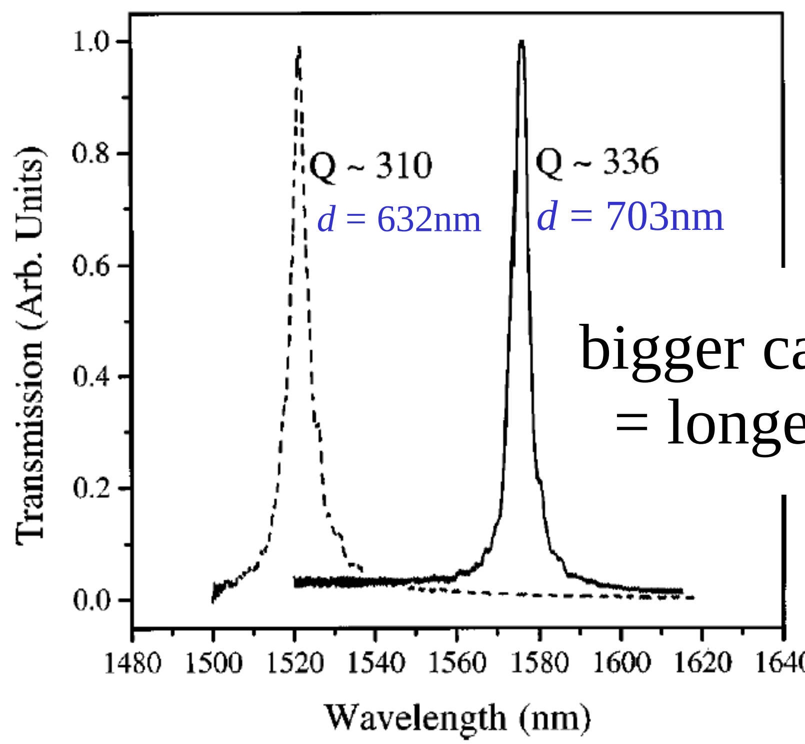
Meanwhile, back in reality…

Air-bridge Resonator: 1d gap + 2d index guiding





[D. J. Ripin *et al.*, *J. Appl. Phys.* **87**, 1578 (2000)]

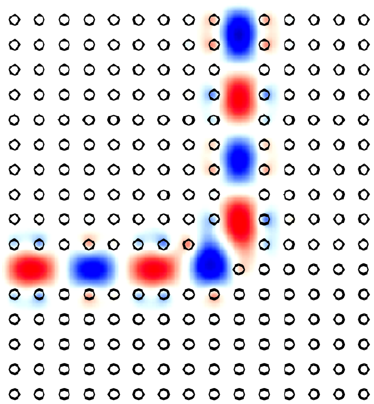


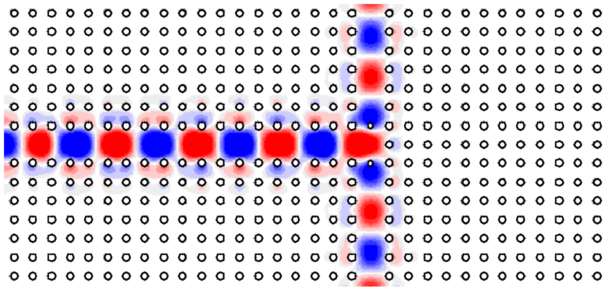
bigger cavity
= longer λ

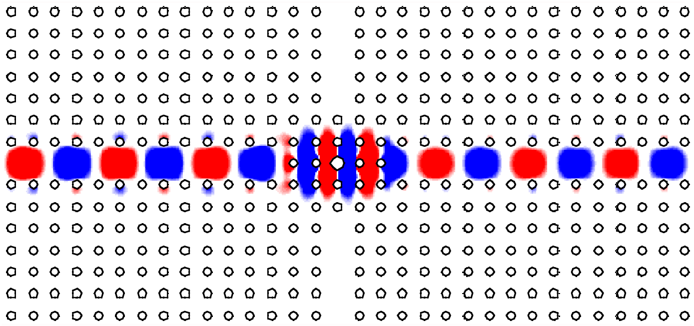
Time for Two Dimensions…

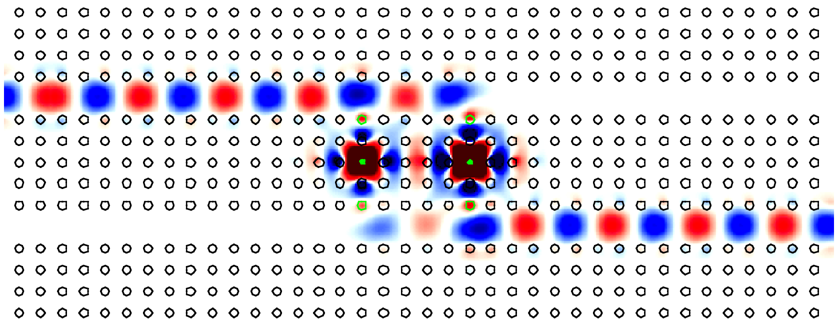
2d is all we really need for many interesting devices

…darn *z* direction!

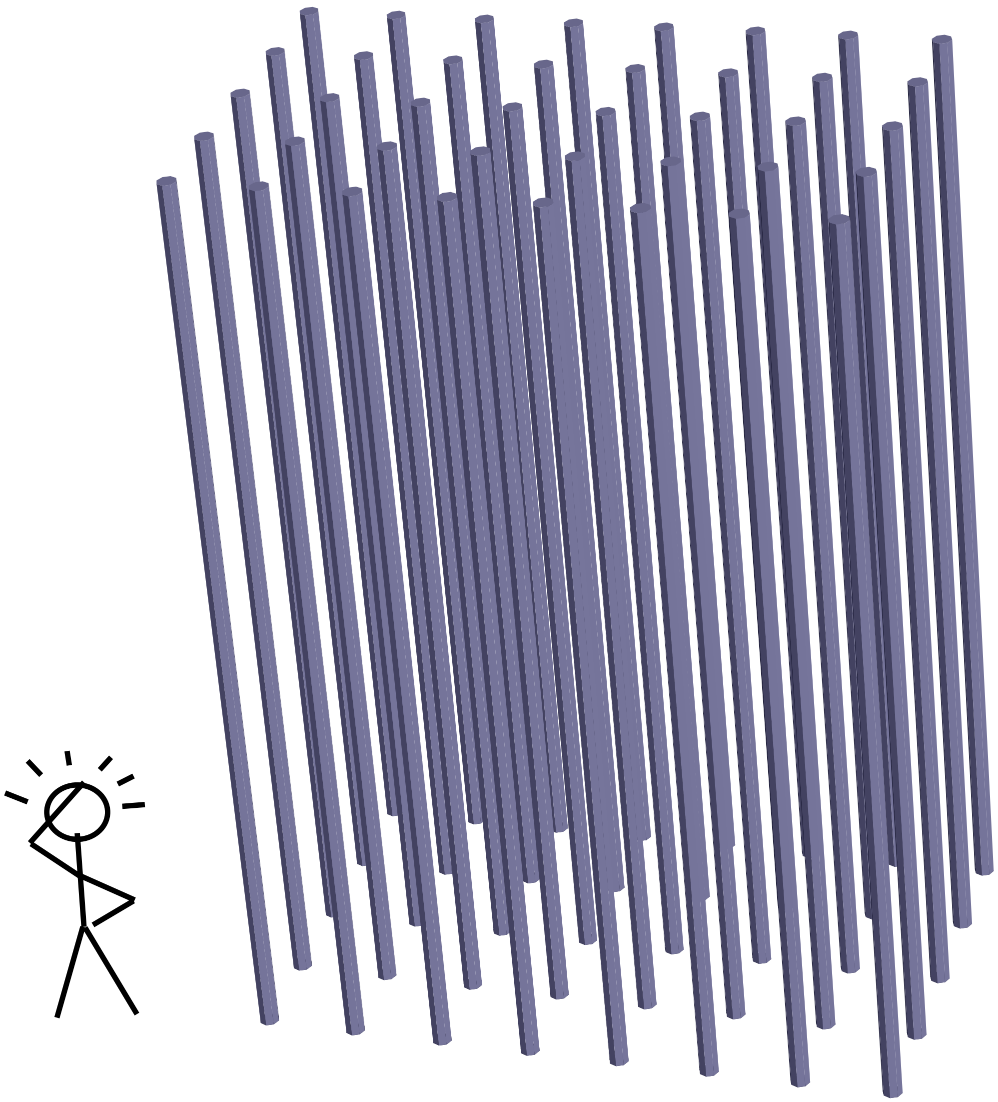








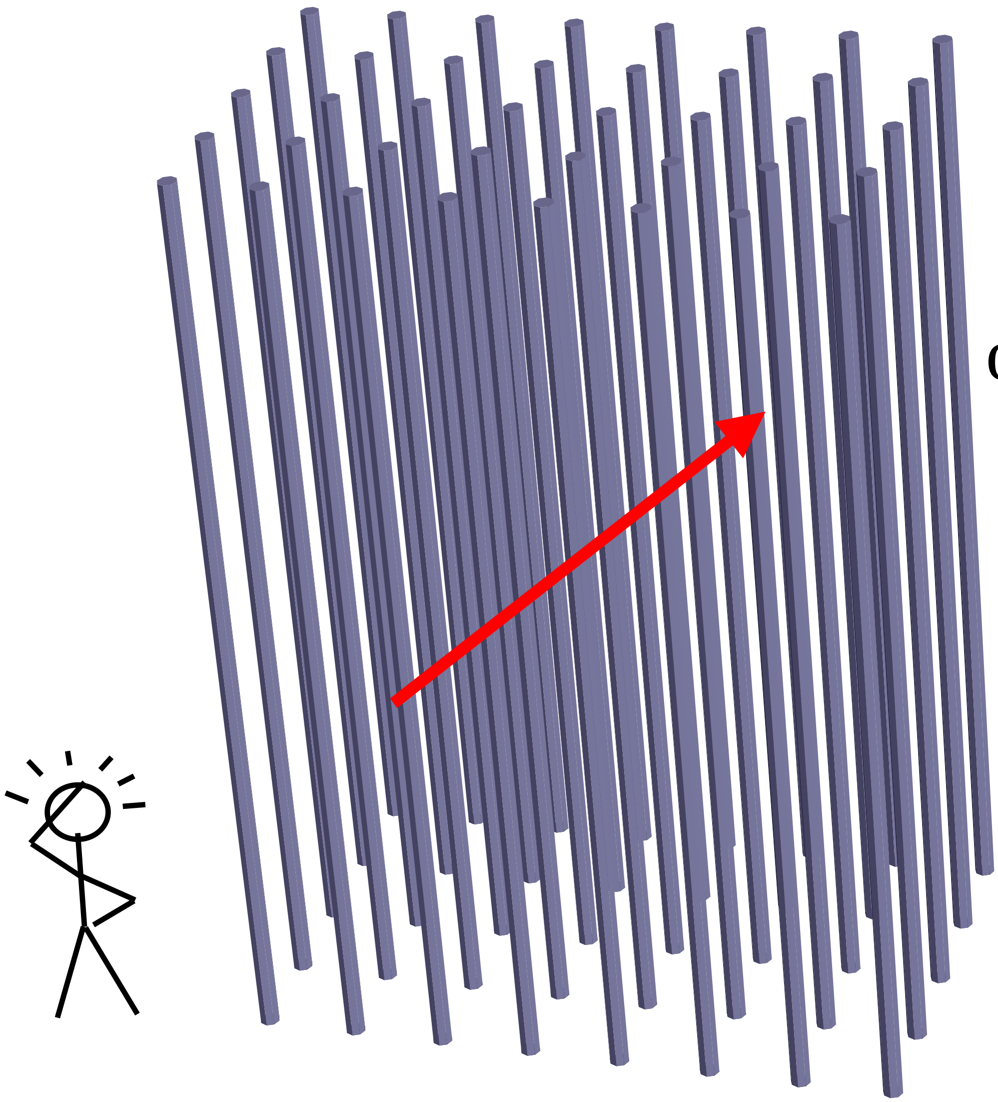
How do we make a 2d bandgap?



Most obvious solution?

make
2d pattern
really tall

How do we make a 2d bandgap?



If height is finite,
we must couple to
out-of-plane wavevectors…

k_z not conserved

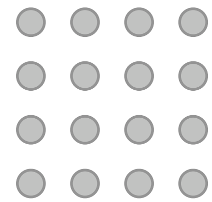
A 2d band diagram in 3d

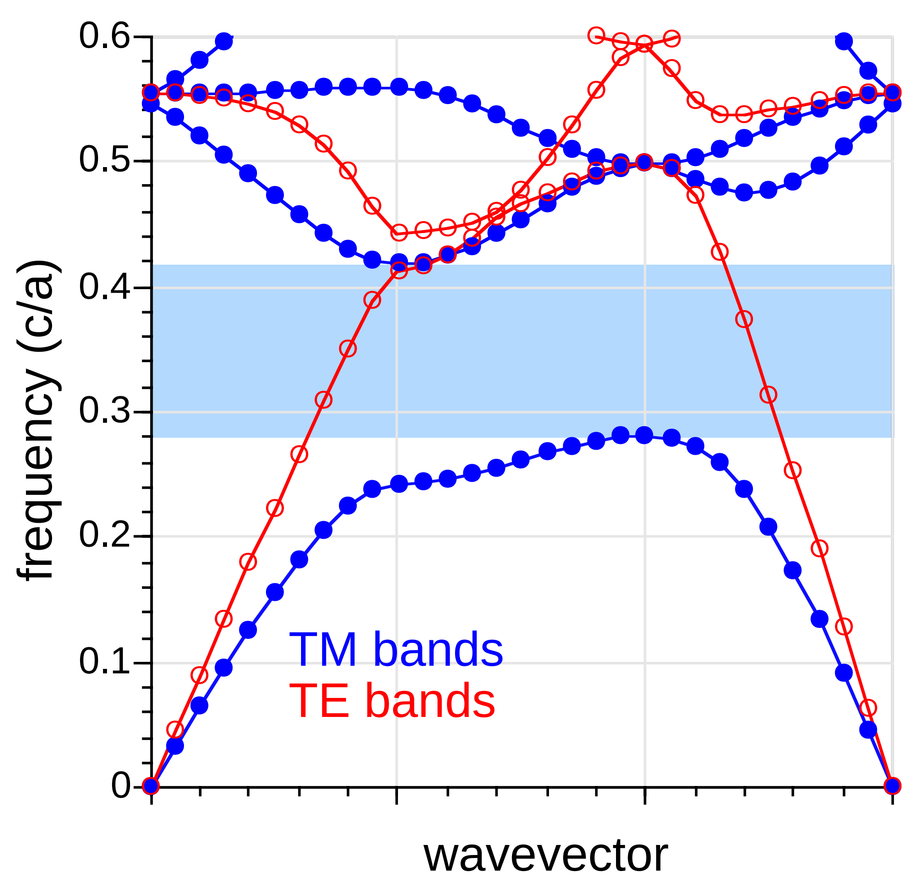
Let’s start with the 2d band diagram.

This is what we’d like to have in 3d, too!

Square Lattice of Dielectric Rods

(ε = 12, r=0.2a)





A 2d band diagram in 3d

Let's start with the 2d band diagram.

This is what we'd like to have in 3d, too!

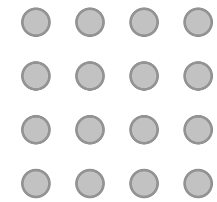
No! When we include **out-of-plane** propagation, we get:

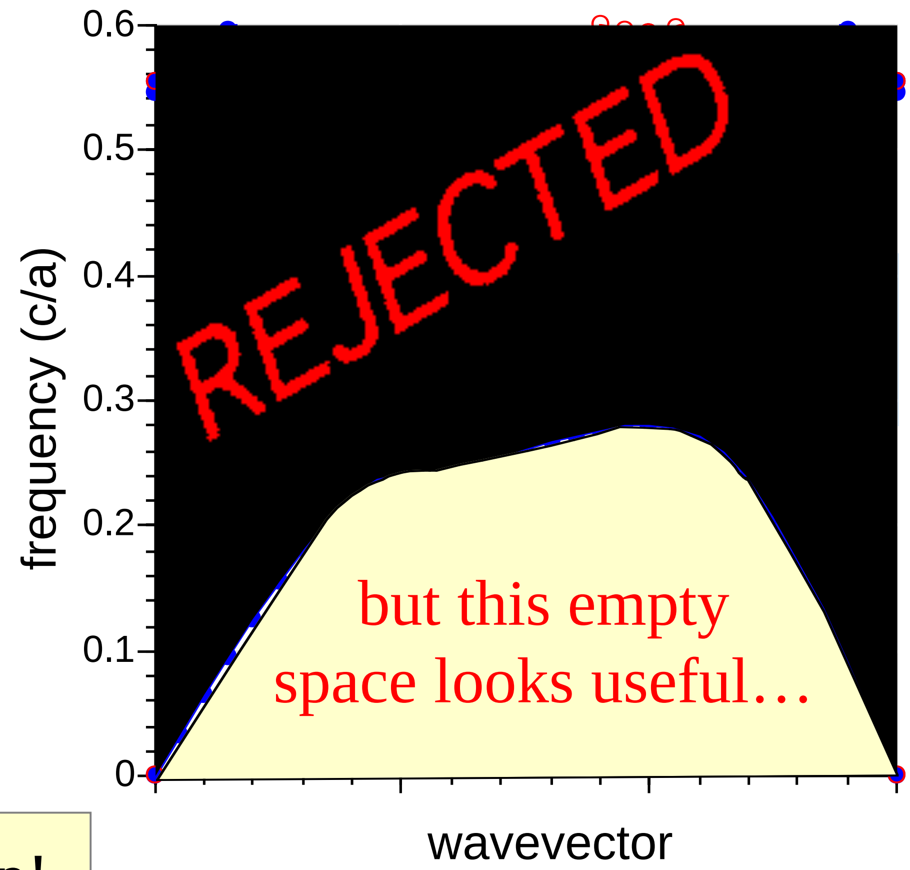
<u>wavevector</u>	<u>frequency</u>
	ω
	ω+δω

projected band diagram fills gap!

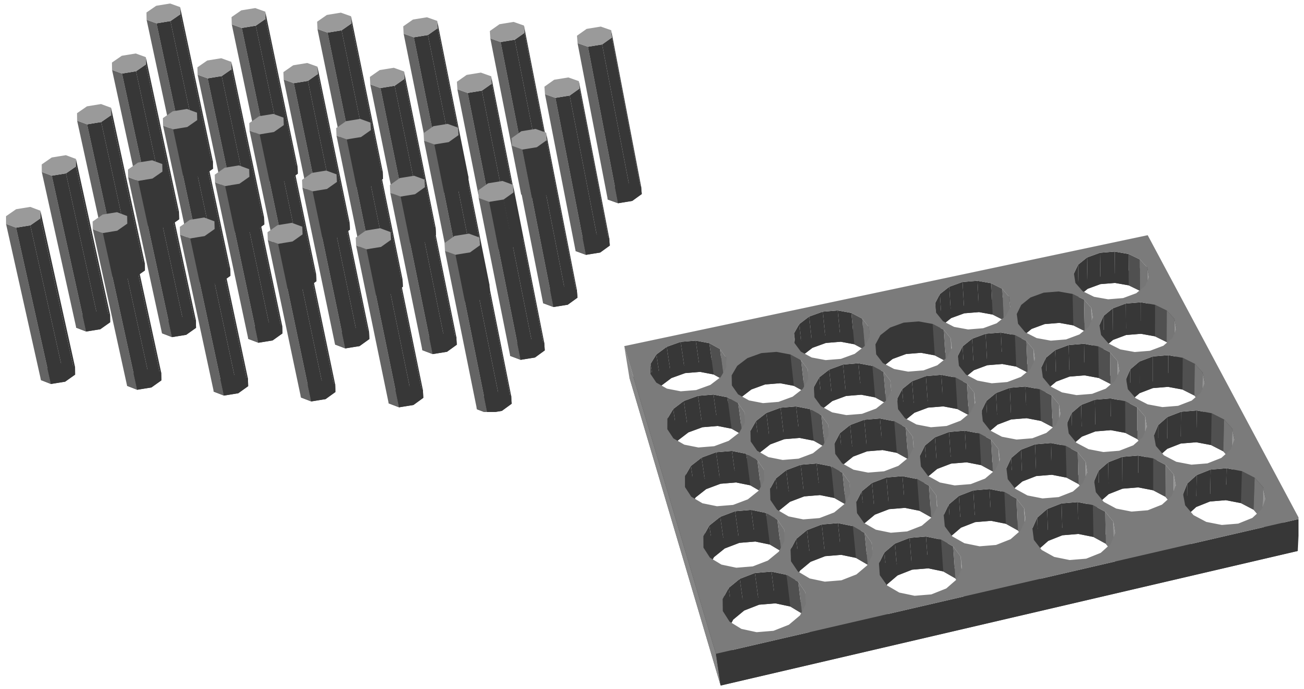
Square Lattice of Dielectric Rods

(ε = 12, r=0.2*a*)





Photonic-Crystal Slabs

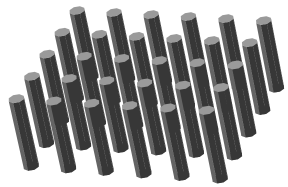


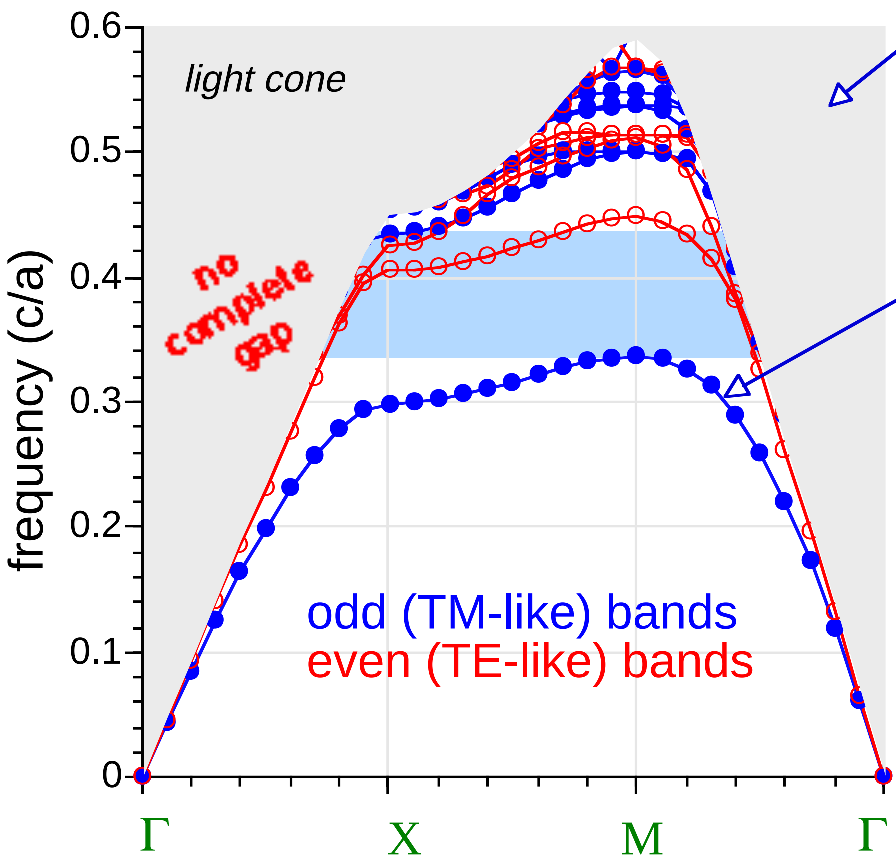
2d photonic bandgap + vertical index guiding

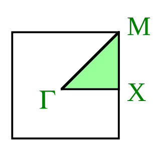
[S. G. Johnson and J. D. Joannopoulos, *Photonic Crystals: The Road from Theory to Practice*]

Rod-Slab Projected Band Diagram

Square Lattice of
Dielectric Rods
(ε = 12, r=0.2*a*, h=2*a*)



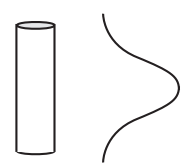




The Light Cone:
All possible states
propagating in the air

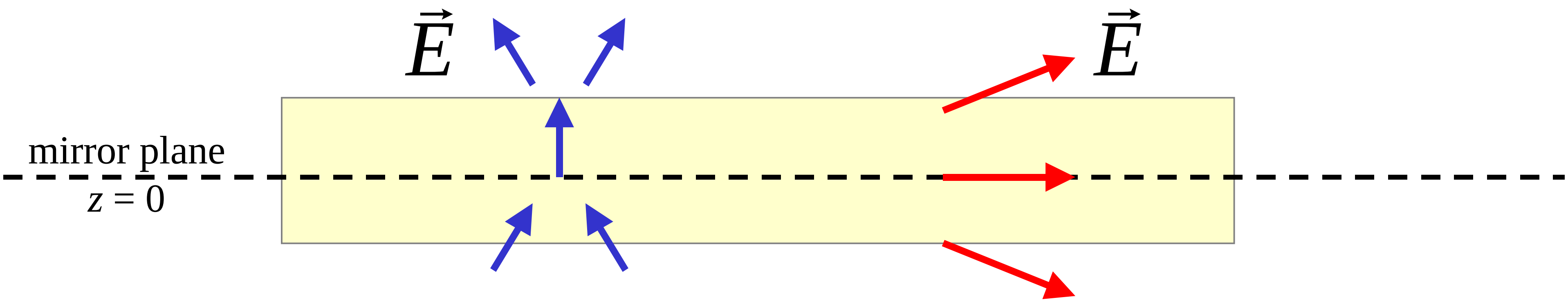
The Guided Modes:
Cannot couple to
the light cone…
—> confined to the slab

Thickness is critical.
Should be about $\lambda/2$
(to have a gap
& be single-mode)



Symmetry in a Slab

2d: TM and TE modes



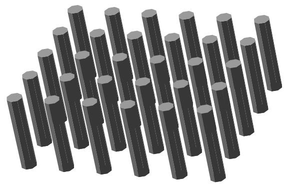
slab: odd (TM-like) and even (TE-like) modes

Like in 2d, there may only be a band gap in one symmetry/polarization

Slab Gaps

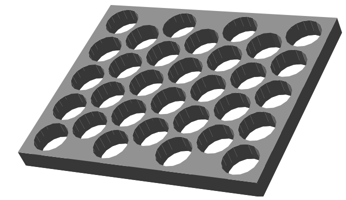
Square Lattice of Dielectric Rods

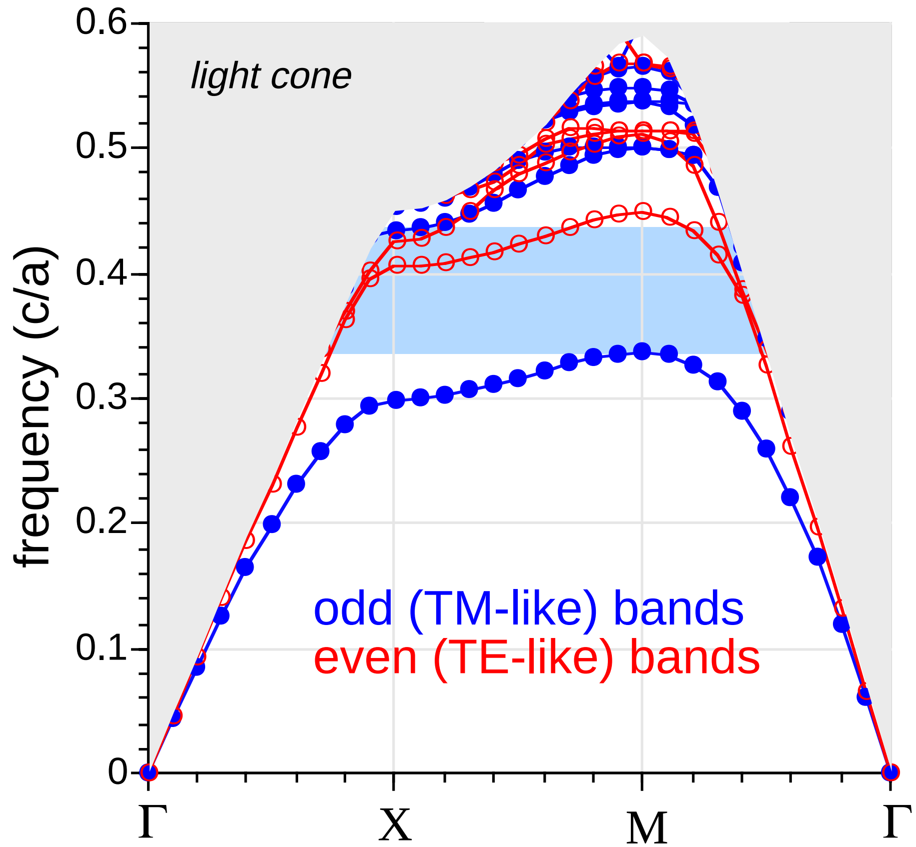
(ε = 12, r=0.2*a*, h=2*a*)



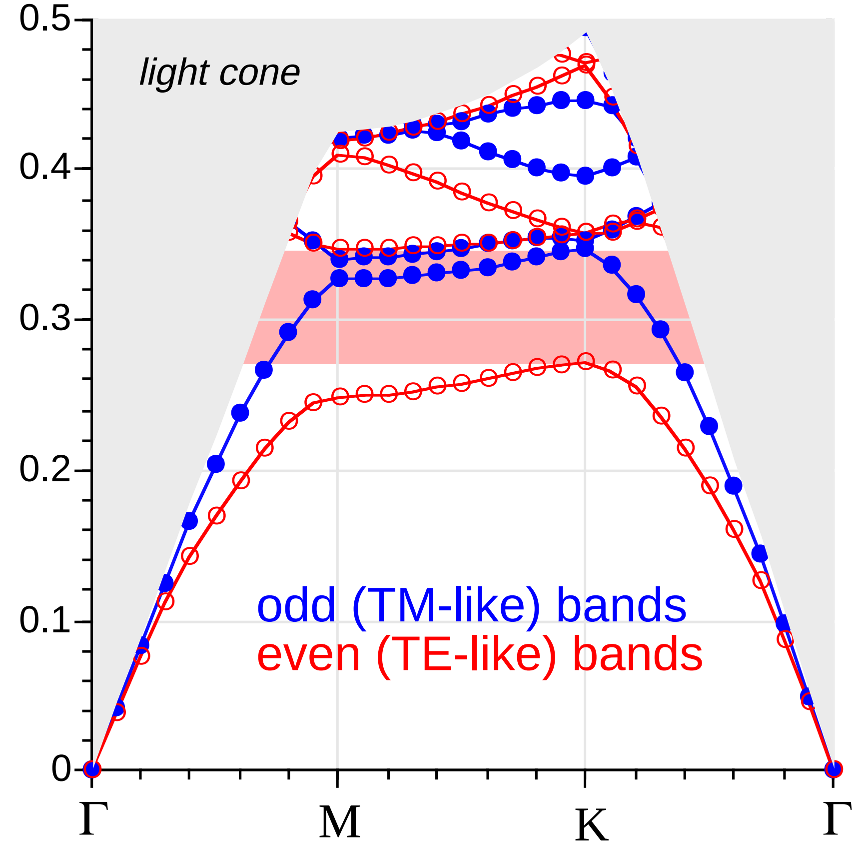
Triangular Lattice of Air Holes

(ε = 12, r=0.3*a*, h=0.5*a*)





TM-like gap

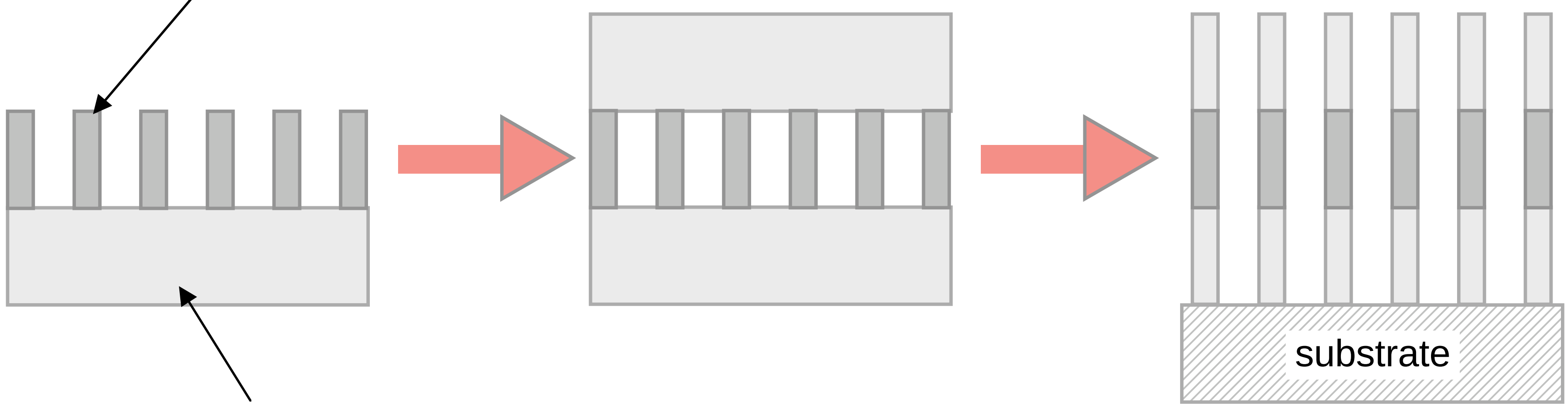


TE-like gap

Substrates, for the Gravity-Impaired

(rods or holes)

superstrate restores symmetry



substrate breaks symmetry:
some even/odd mixing "kills" gap

BUT

with strong confinement

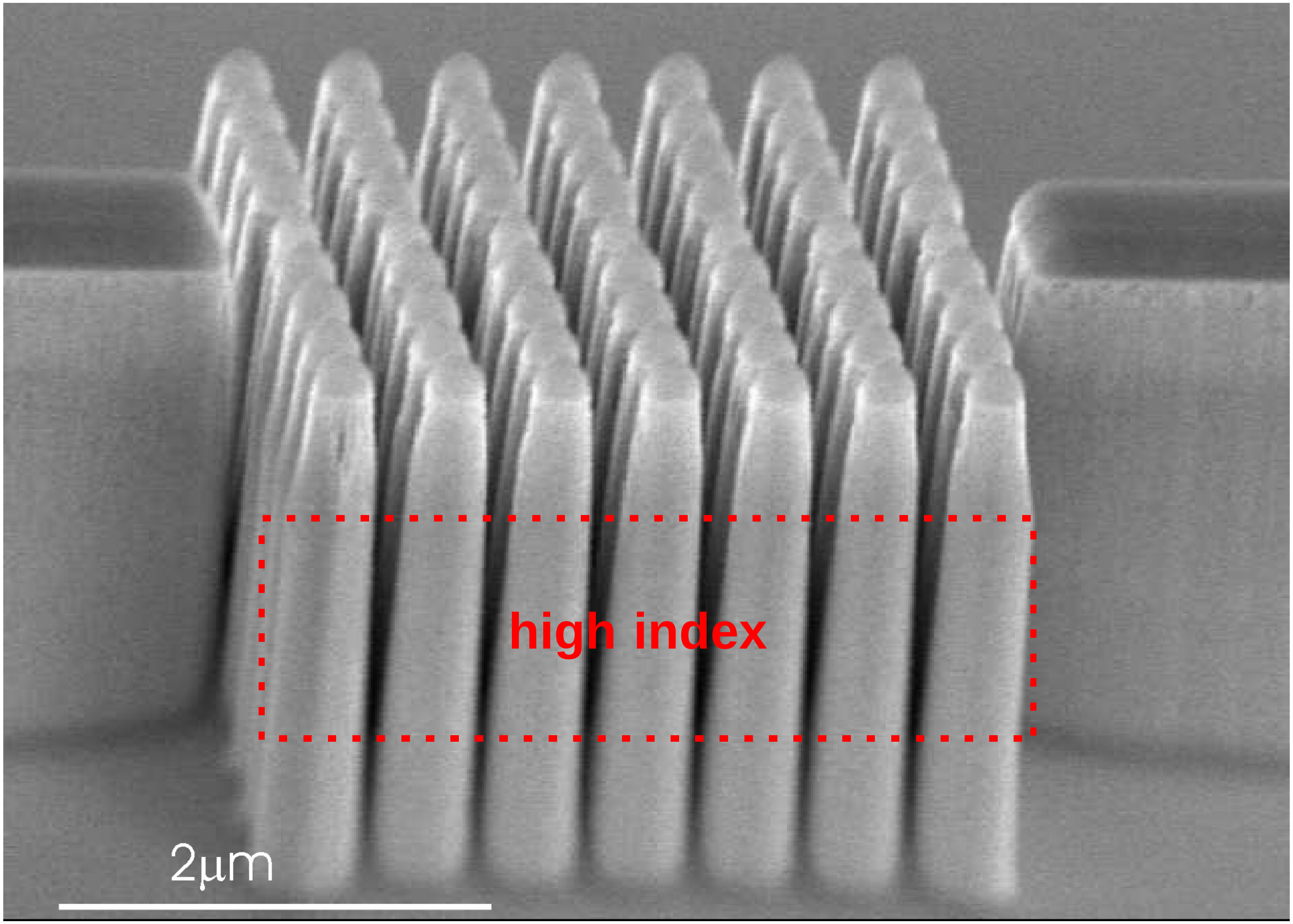
(high index contrast)

mixing can be weak

"extruded" substrate
= stronger confinement

(less mixing even
without superstrate

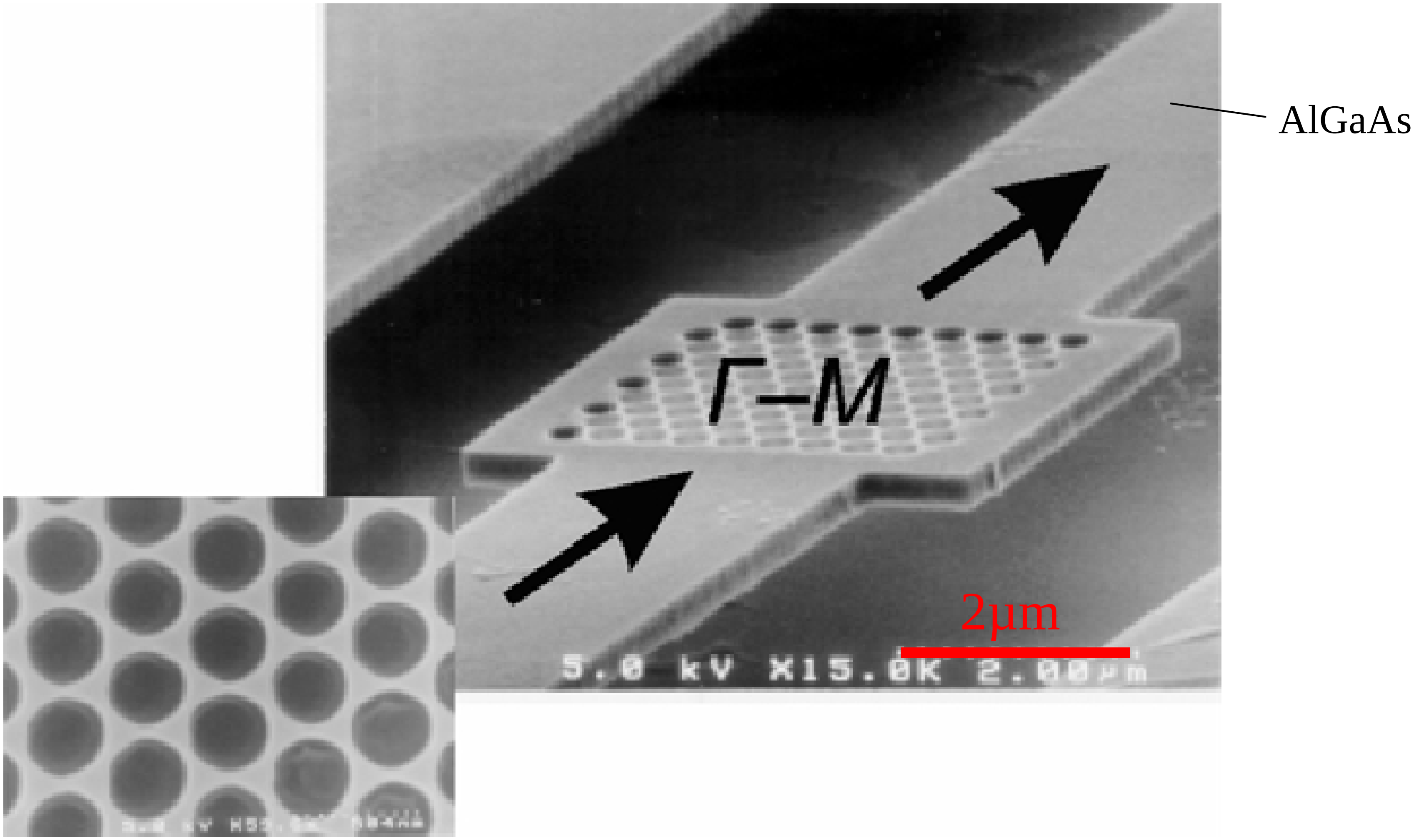
Extruded Rod Substrate



S. Assefa, L. A. Kolodziejski

Air-membrane Slabs

who needs a substrate?

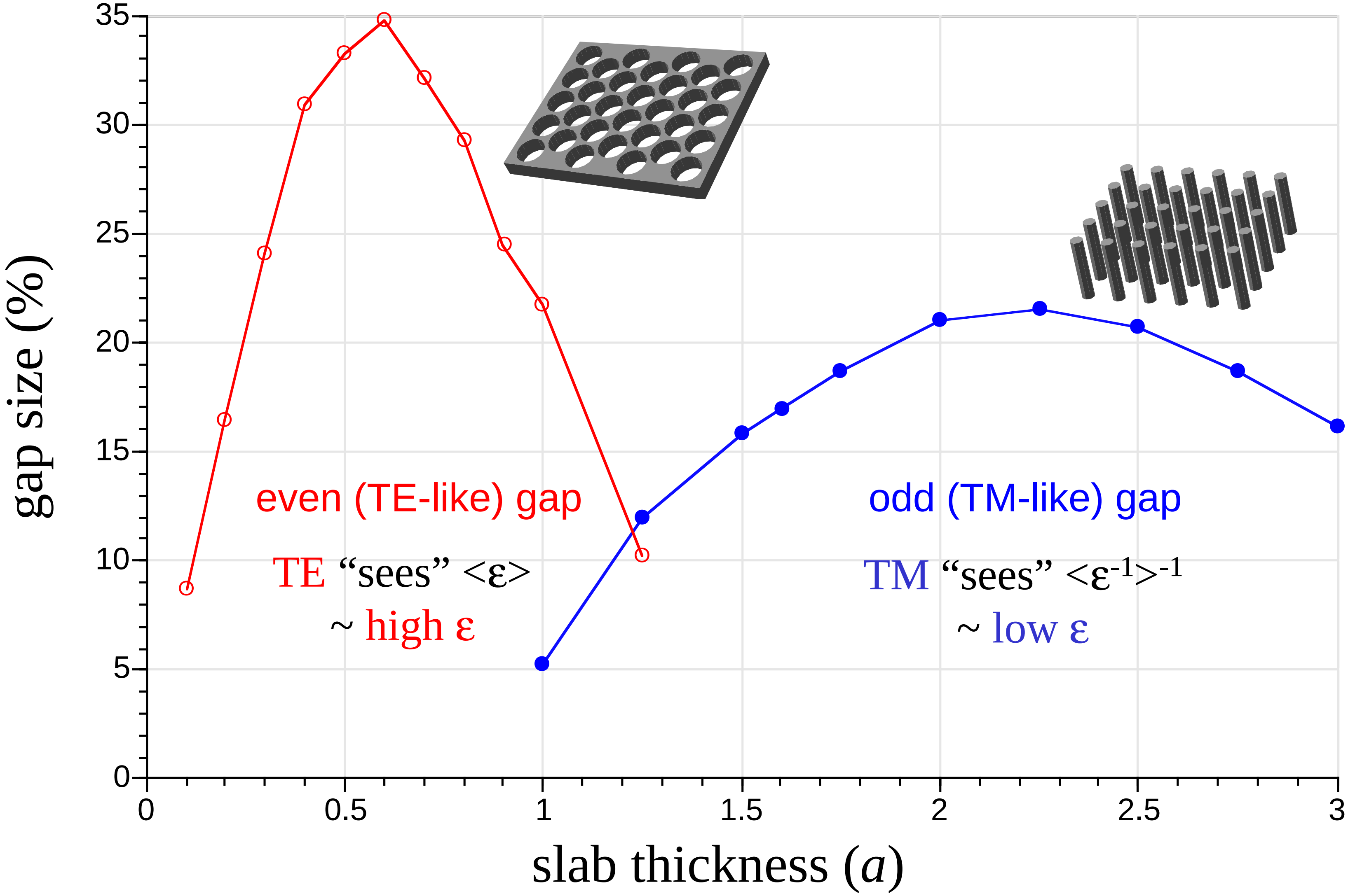


[N. Carlsson *et al*., *Opt. Quantum Elec*. **34**, 123 (2002)]

Optimal Slab Thickness

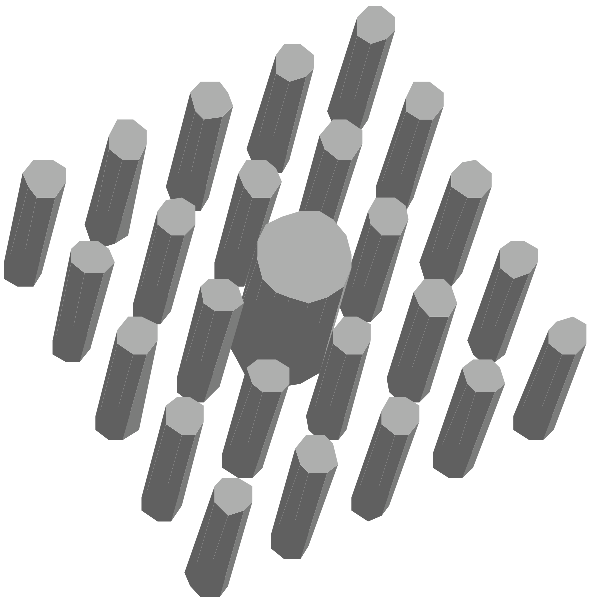
~ λ/2, but λ/2 in what material?

effective medium theory: effective ε depends on polarization

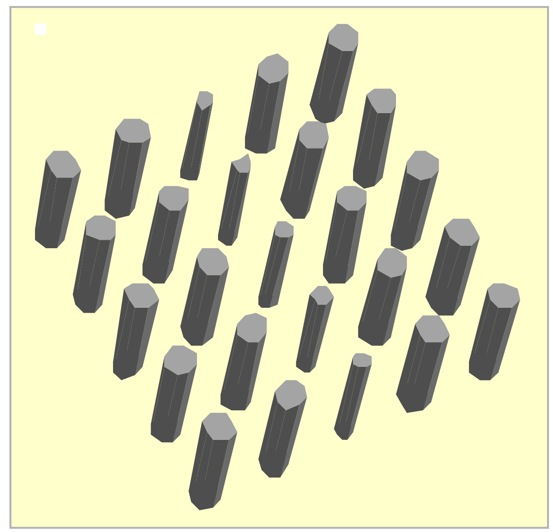


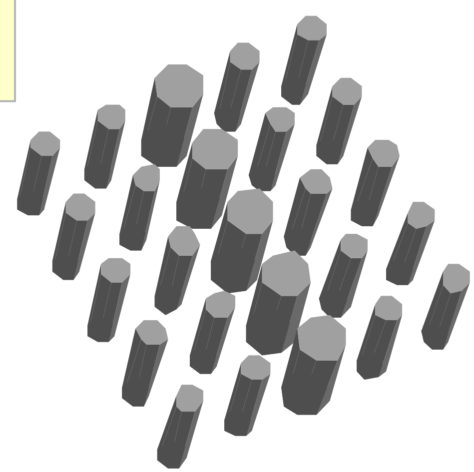
Photonic-Crystal Building Blocks

point defects
(cavities)

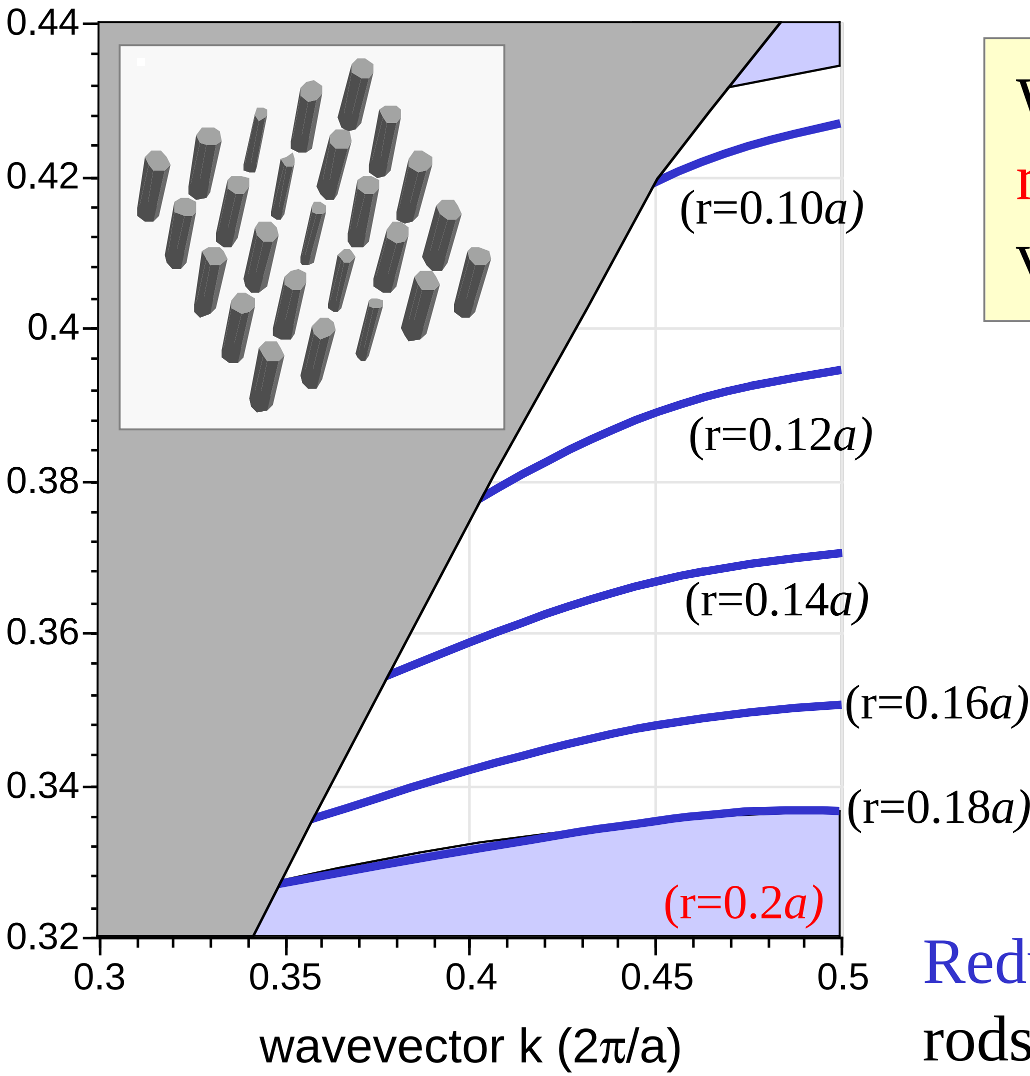


line defects
(waveguides)





A Reduced-Index Waveguide

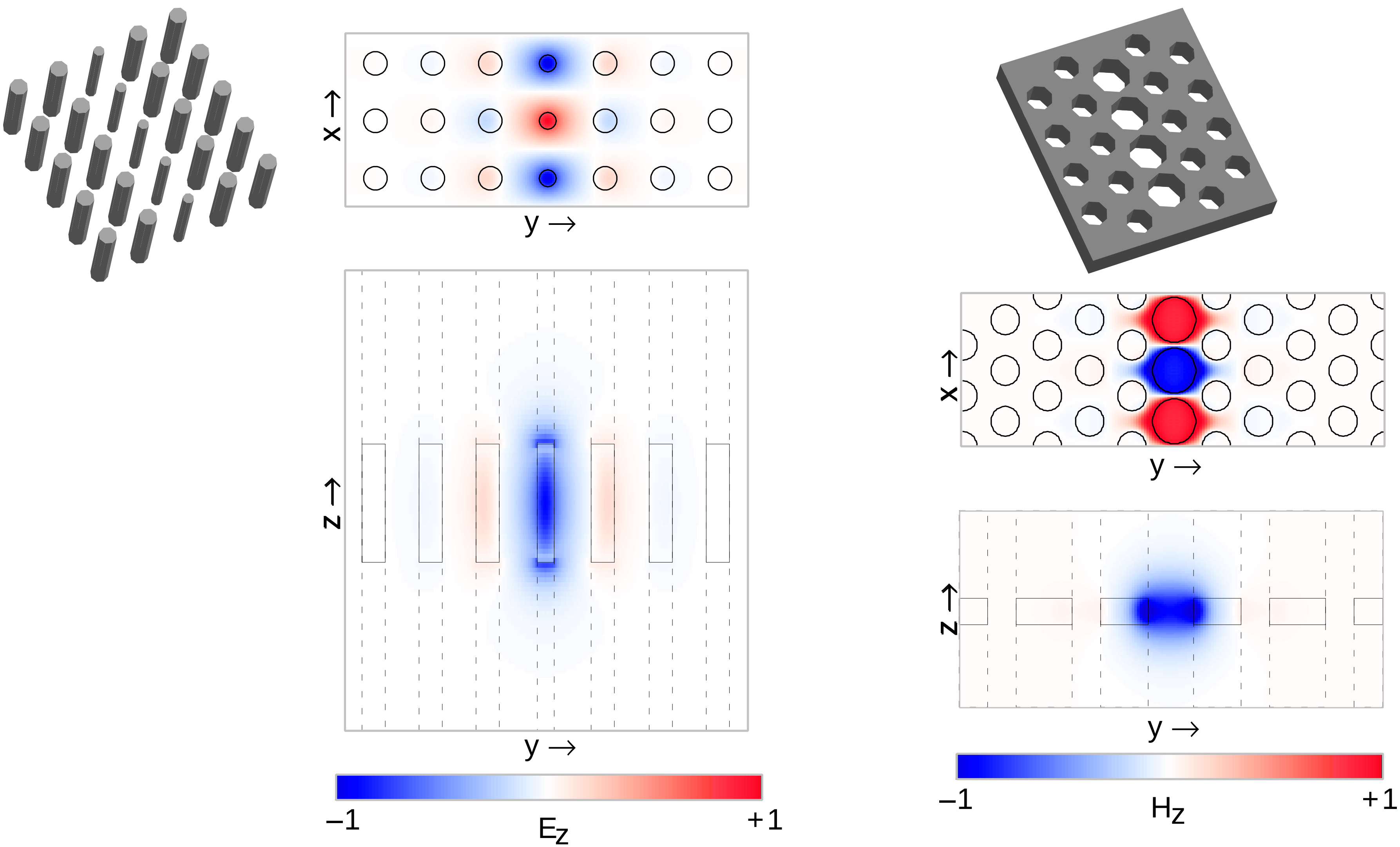


We *cannot* completely remove the rods—no vertical confinement!

Still have conserved wavevector—under the light cone, no radiation

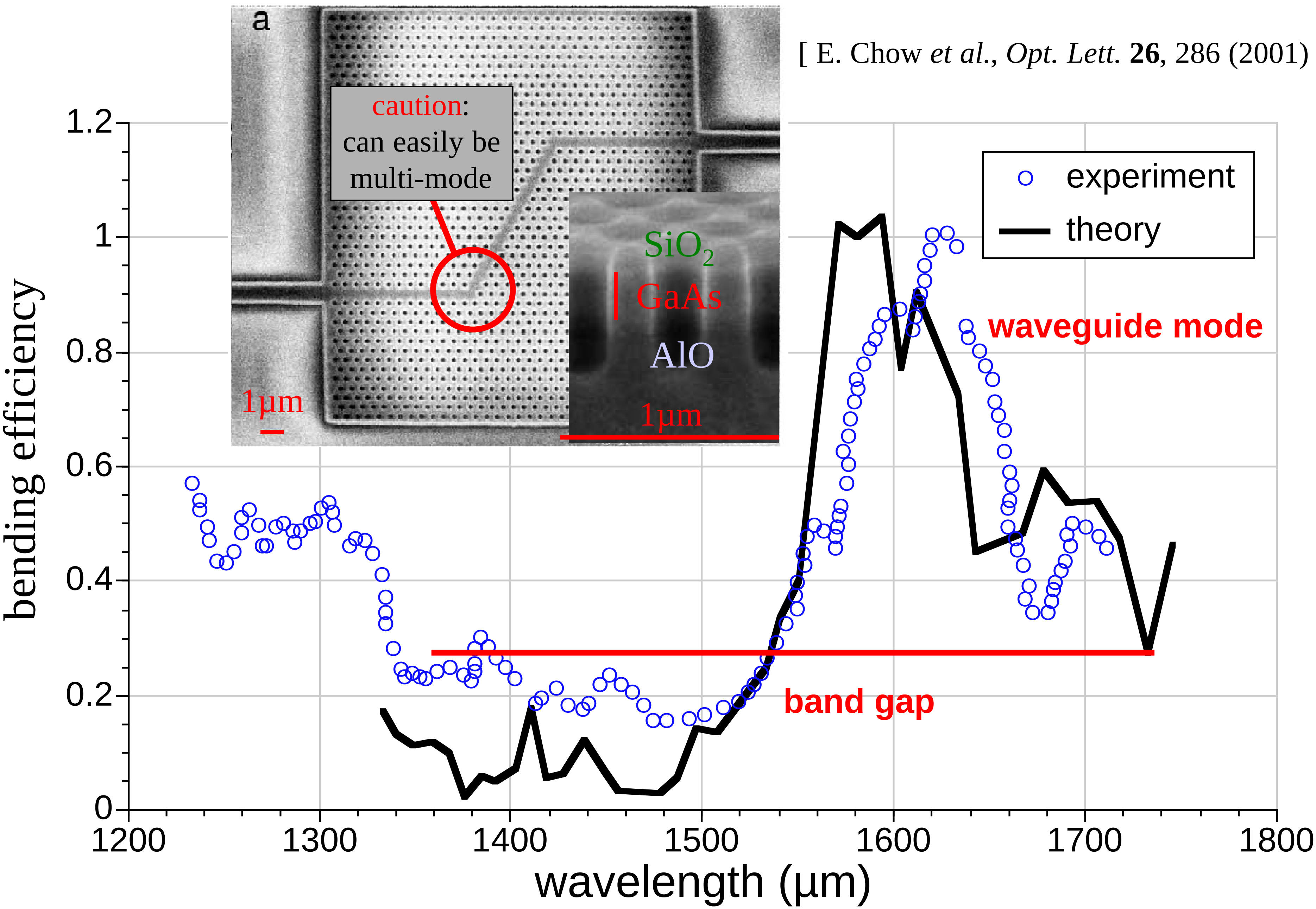
Reduce the radius of a row of rods to "trap" a waveguide mode in the gap.

Reduced-Index Waveguide Modes



Experimental Waveguide & Bend

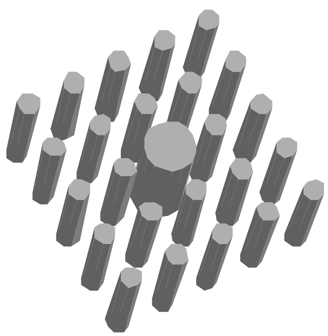
[E. Chow *et al.*, *Opt. Lett.* **26**, 286 (2001)]

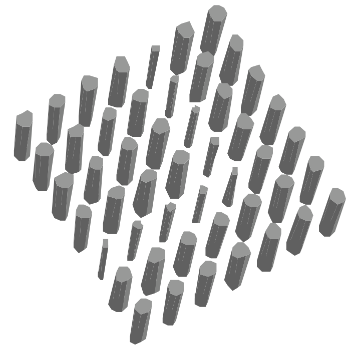


Inevitable Radiation Losses

whenever translational symmetry is broken

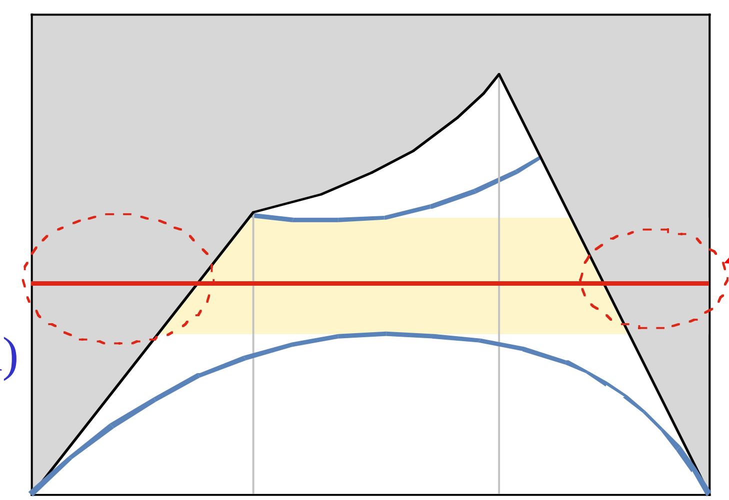
e.g. at cavities, waveguide bends, disorder…





coupling to light cone
= radiation losses

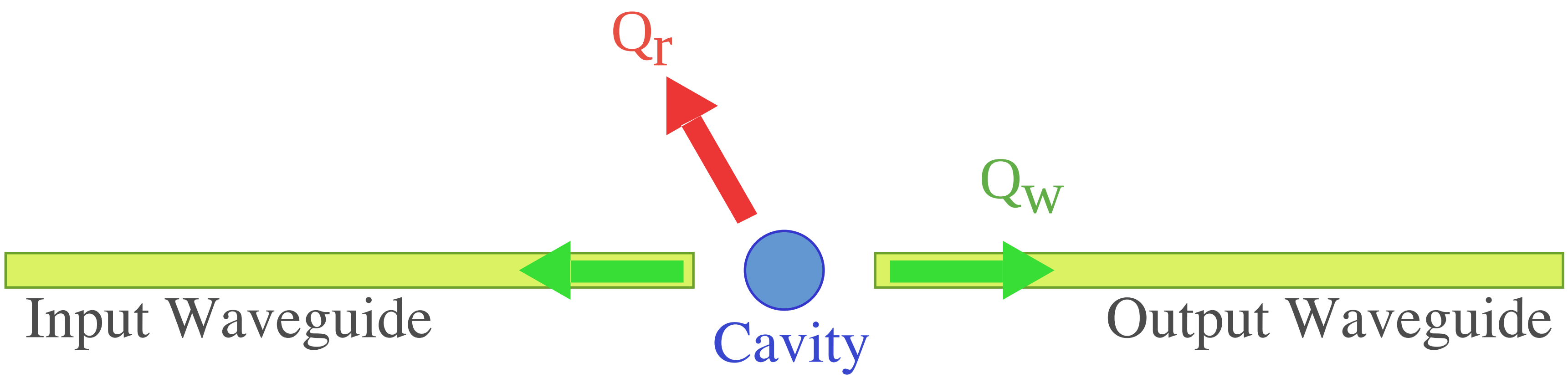
ω
(conserved)



k is *no longer* conserved!

All Is Not Lost

A simple model device (filters, bends, …):



$$\frac{1}{Q} = \frac{1}{Q_r} + \frac{1}{Q_w}$$

Q = lifetime/period
= frequency/bandwidth

We want: $Q_r \gg Q_w$

$1 - \text{transmission} \sim 2Q / Q_r$

worst case: high-Q (narrow-band) cavities

Semi-analytical losses

A low-loss strategy:

Make field inside defect small = delocalize mode

Make defect weak = delocalize mode

$$\vec{E}(\vec{x}) = \int_{defect} \overleftrightarrow{G}_{\omega}(\vec{x}, \vec{x}') \cdot \vec{E}(\vec{x}') \cdot \Delta\varepsilon(\vec{x}')$$

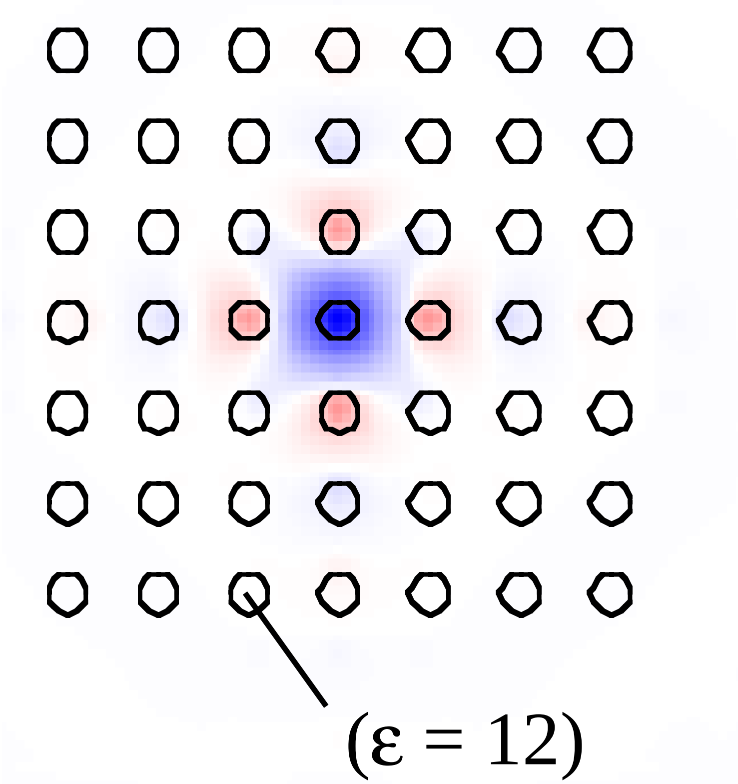
far-field (radiation)

Green's function (defect-free system)

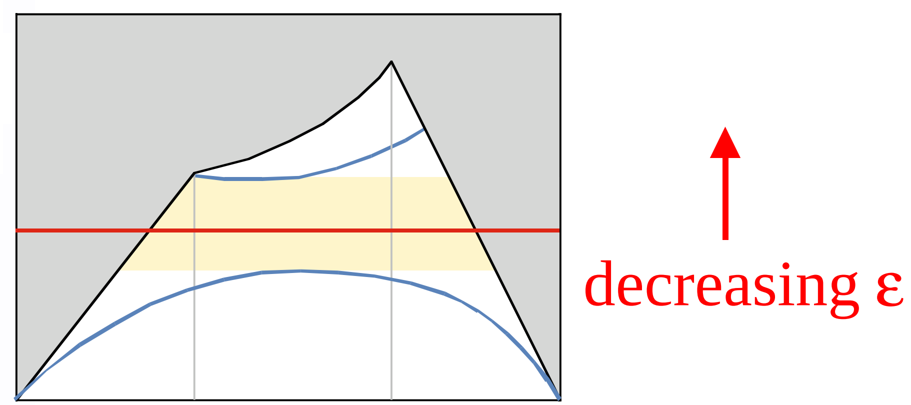
near-field (cavity mode)

defect

Monopole Cavity in a Slab

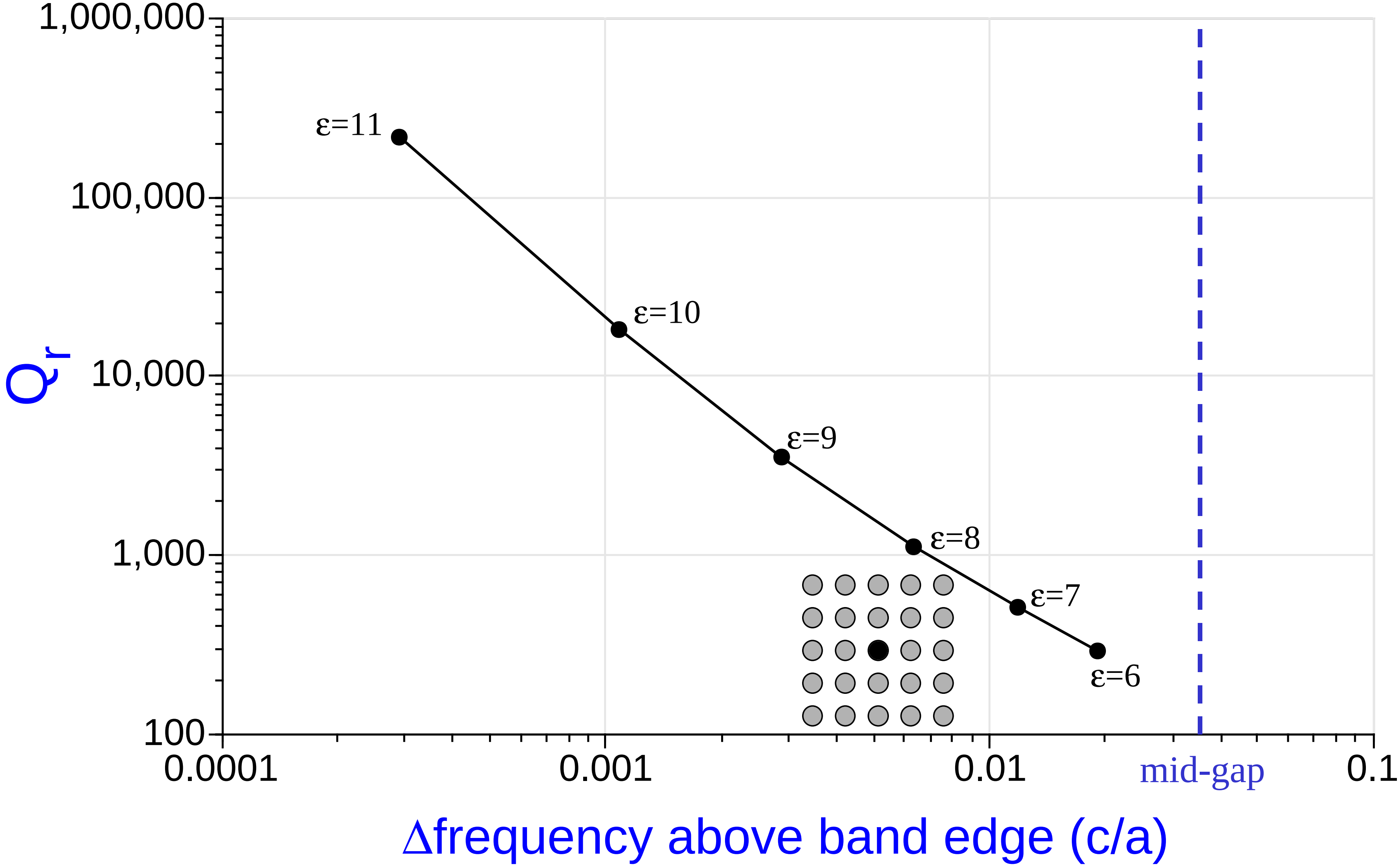


Lower the ε of a single rod: push up a monopole (singlet) state.

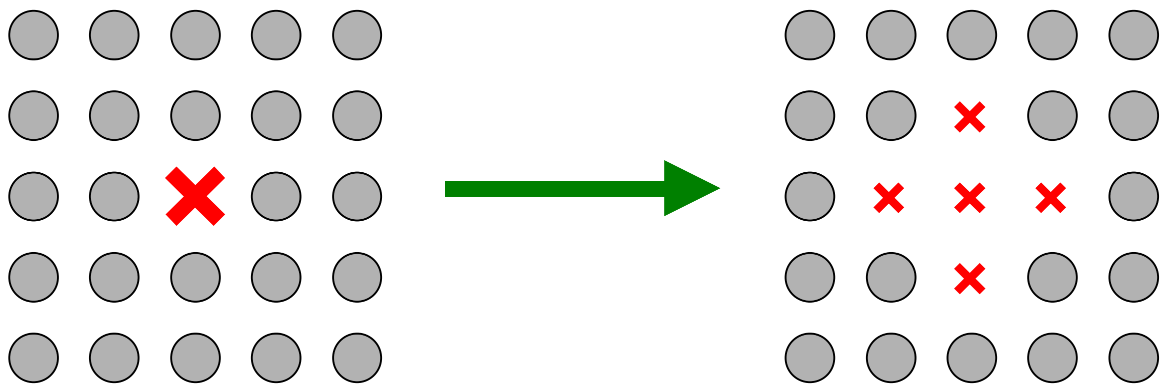


Use small Δε: delocalized in-plane, & high-Q (we hope)

Delocalized Monopole Q



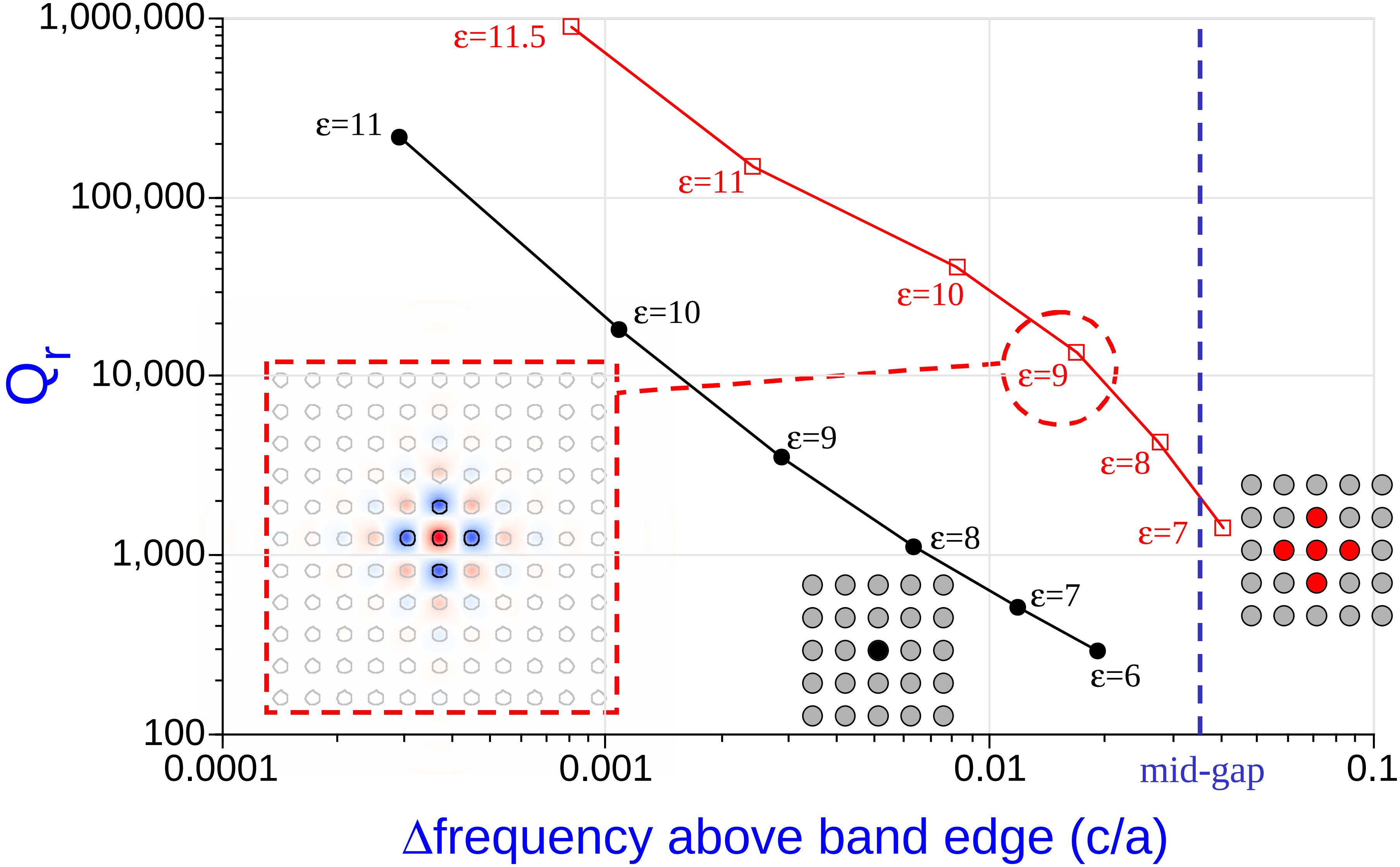
Super-defects



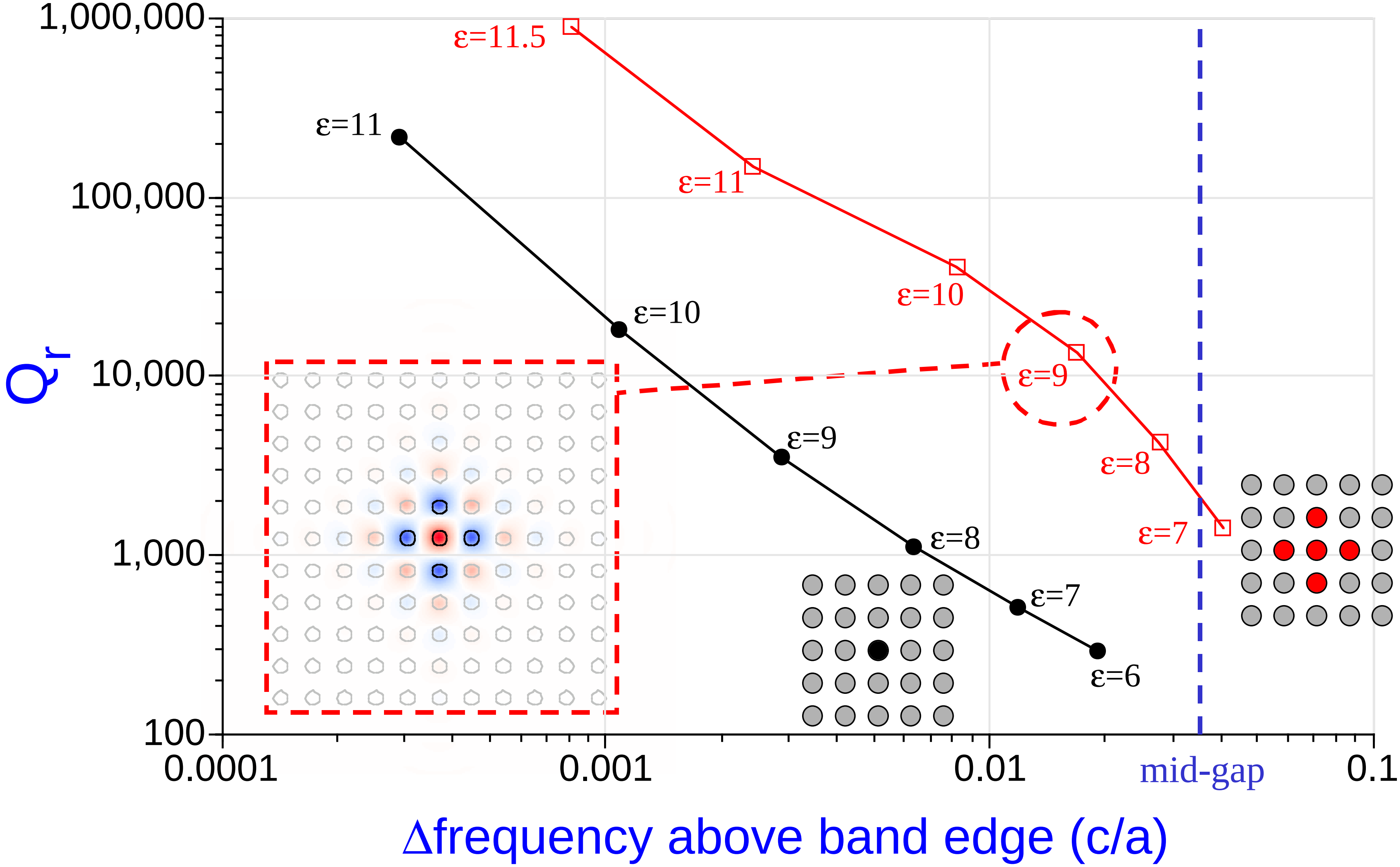
Weaker defect with more unit cells.

More delocalized
at the same point in the gap
(*i.e.* at same bulk decay rate)

Super-Defect vs. Single-Defect Q

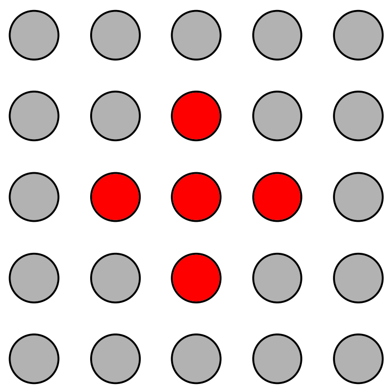


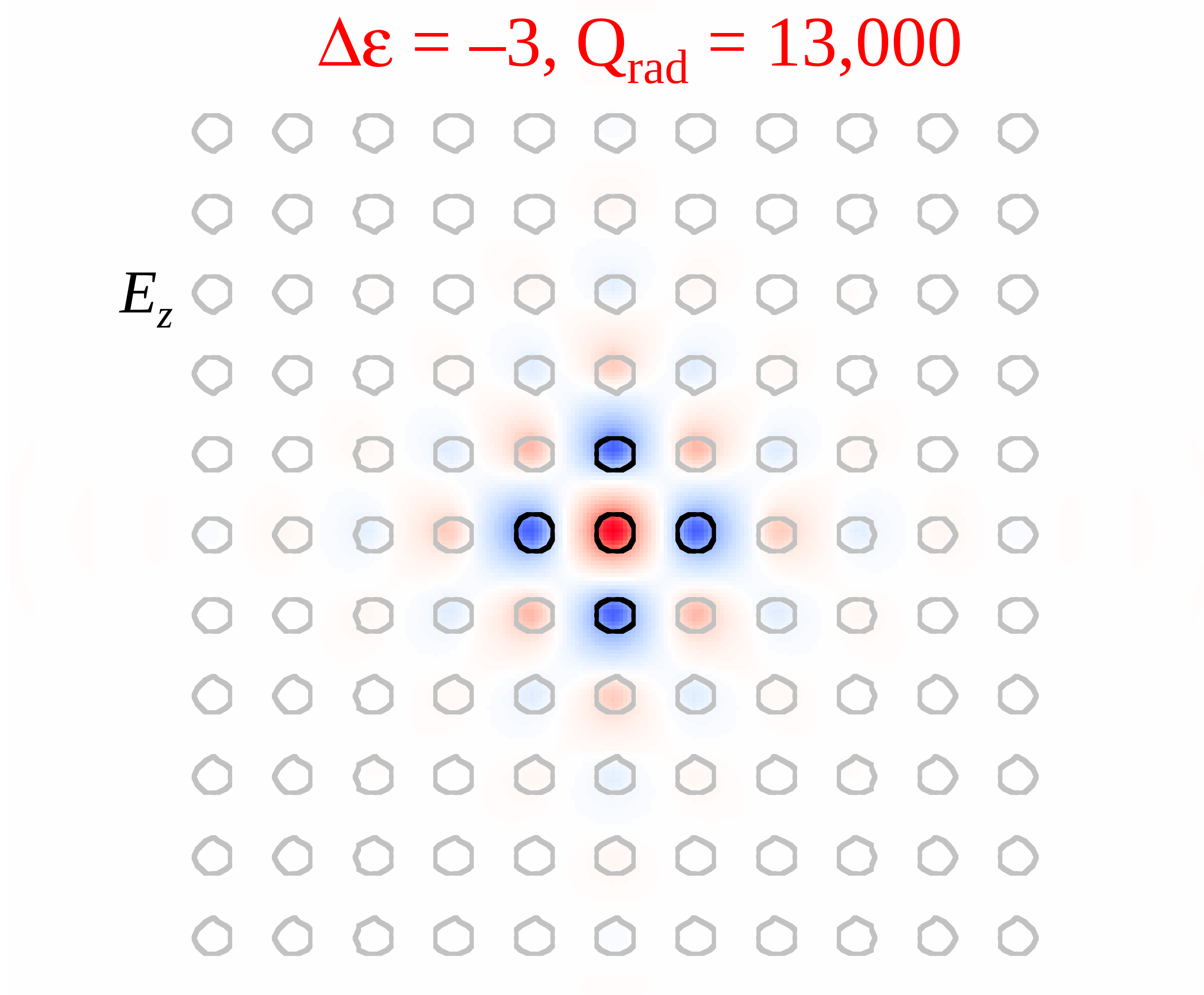
Super-Defect vs. Single-Defect Q



Super-Defect State

(cross-section)

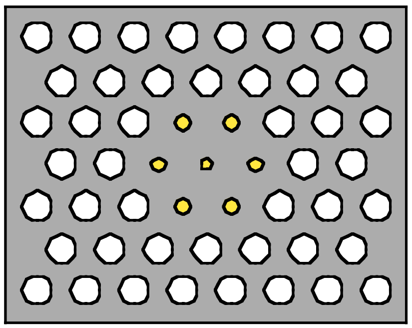

(super defect)



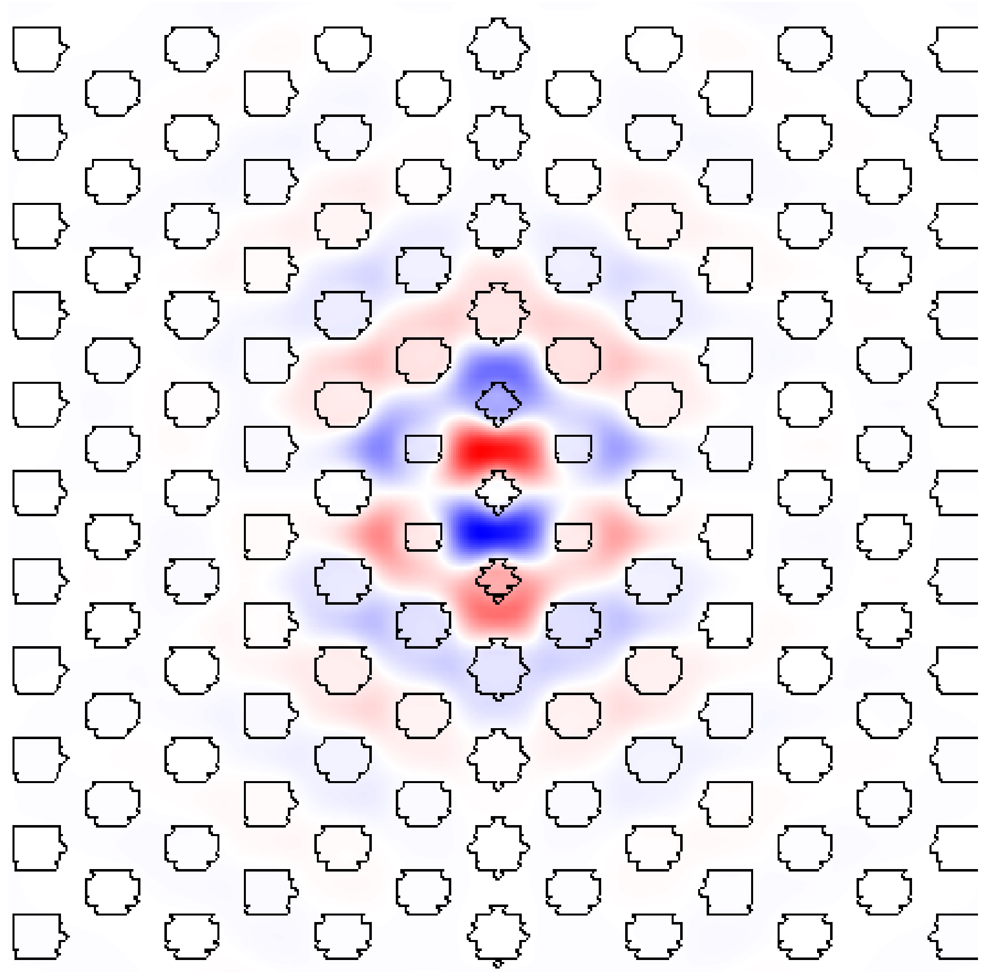
still ~localized: *In-plane* $Q_{||}$ is > 50,000 for only 4 bulk periods

Q = 2500
near mid-gap (Δfreq = 0.03)

Hole Slab
ε=11.56
period *a*, radius 0.3*a*
thickness 0.5*a*



Reduce radius of
7 holes to 0.2*a*

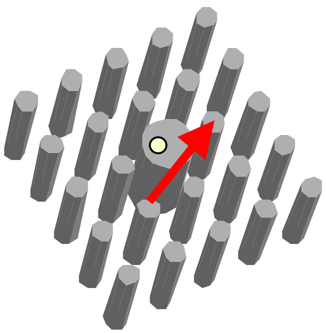


Very robust to roughness
(note pixellization, *a* = 10 pixels).

How do we compute Q?

(via 3d FDTD [finite-difference time-domain] simulation)

1



excite cavity with dipole source

(broad bandwidth, *e.g.* Gaussian pulse)

… monitor field at some point ∘

…extract frequencies, decay rates via signal processing (FFT is suboptimal)

[V. A. Mandelshtam, *J. Chem. Phys.* **107**, 6756 (1997)]

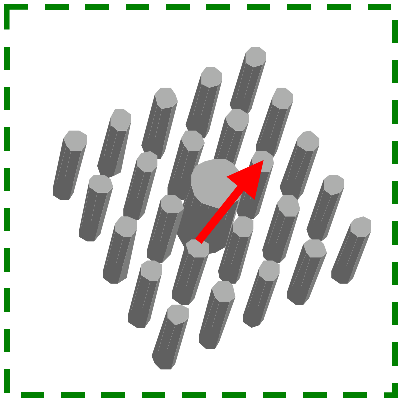
Pro: no *a priori* knowledge, get all ω's and Q's at once

Con: no separate Q_w/Q_r, Q > 500,000 hard,
mixed-up field pattern if multiple resonances

How do we compute Q?

(via 3d FDTD [finite-difference time-domain] simulation)

2



excite cavity with
narrow-band dipole source
(*e.g.* temporally broad Gaussian pulse)

— source is at ω_0 resonance,
which must already be known (via 1)

…measure outgoing power P and energy U

$$Q = \omega_0 U / P$$

Pro: separate Q_w/Q_r, arbitrary Q, also get field pattern

Con: requires separate run 1 to get ω_0,
long-time source for closely-spaced resonances

Can we increase Q

without delocalizing?

Semi-analytical losses

Another low-loss strategy:

exploit cancellations from sign oscillations

$$\vec{E}(\vec{x}) = \int_{defect} \overset{\leftrightarrow}{G}_\omega(\vec{x},\vec{x}') \cdot \vec{E}(\vec{x}') \cdot \Delta\varepsilon(\vec{x}')$$

far-field (radiation)

Green's function (defect-free system)

near-field (cavity mode)

defect

Need a more compact representation

Cannot cancel infinitely many $\mathbf{E}(x)$ integrals

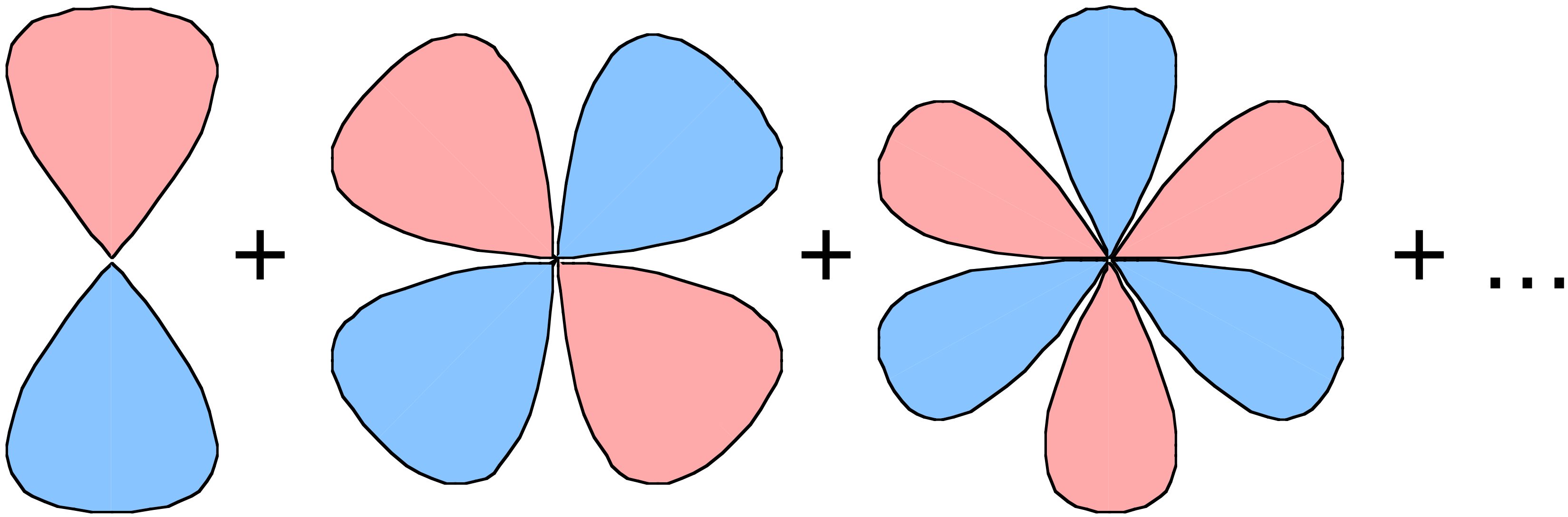
Radiation pattern from localized source…

— use multipole expansion
& cancel largest moment

Multipole Expansion

[Jackson, *Classical Electrodynamics*]

radiated field =



dipole quadrupole hexapole

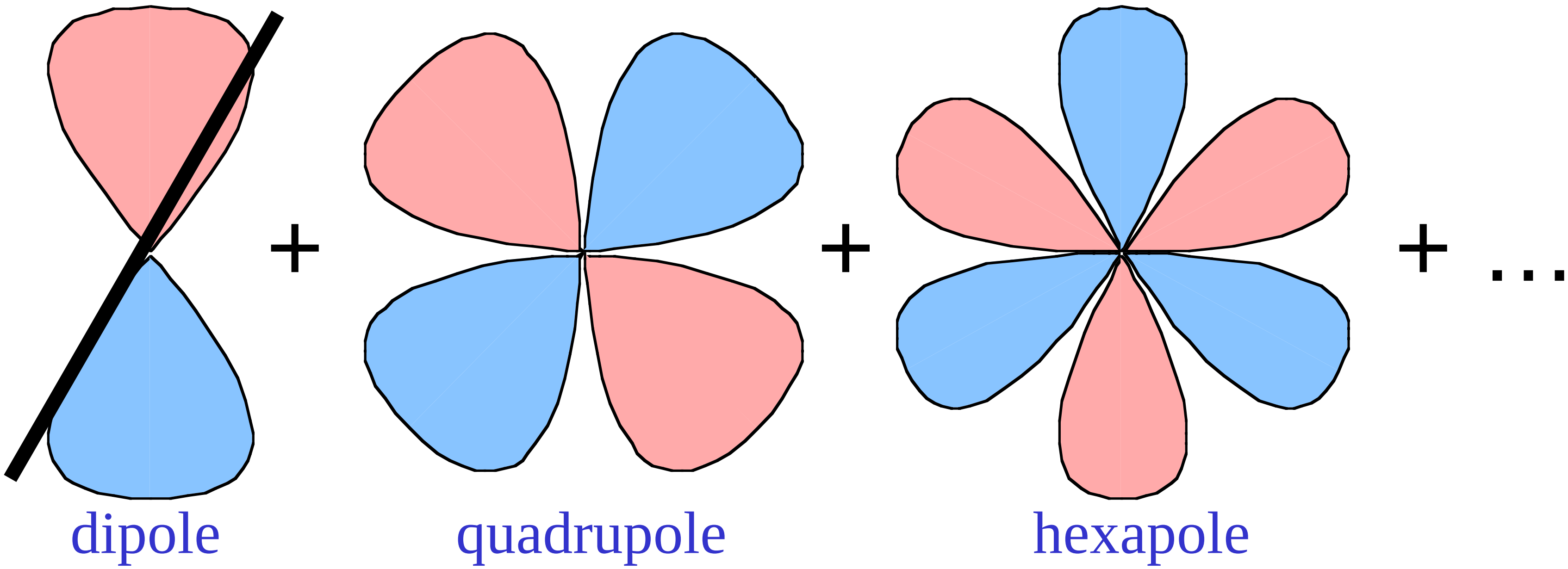
Each term’s strength = single integral over near field

…one term is cancellable by tuning one defect parameter

Multipole Expansion

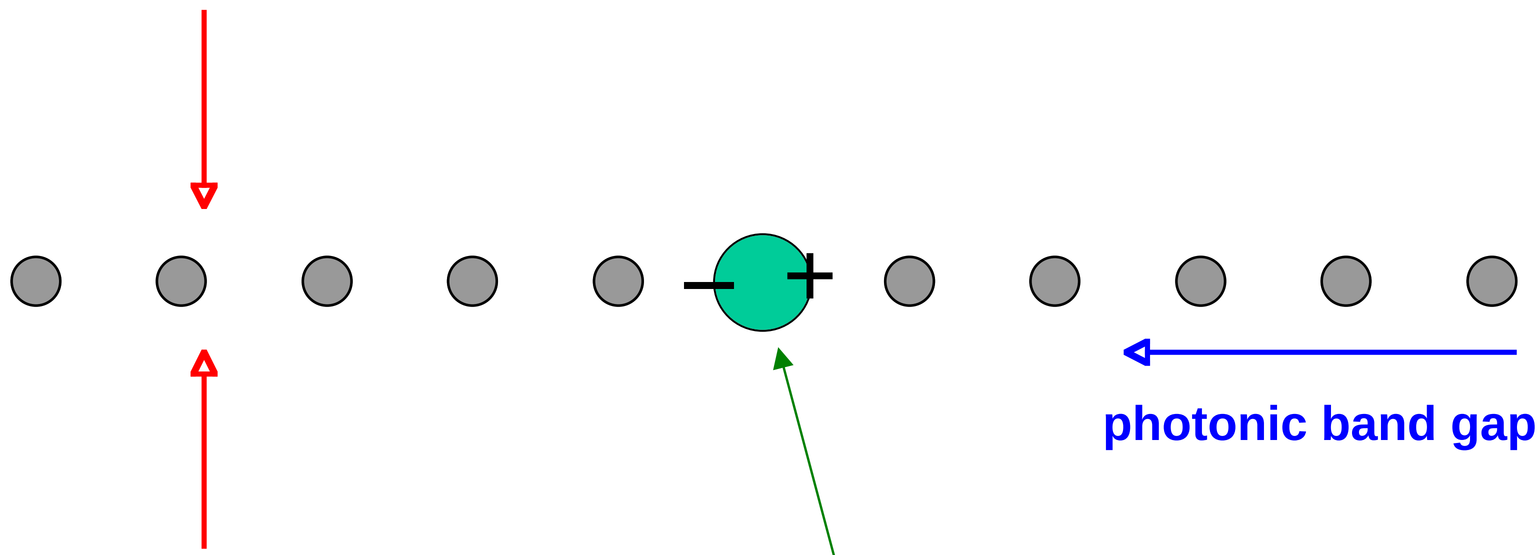
[Jackson, *Classical Electrodynamics*]

radiated field =



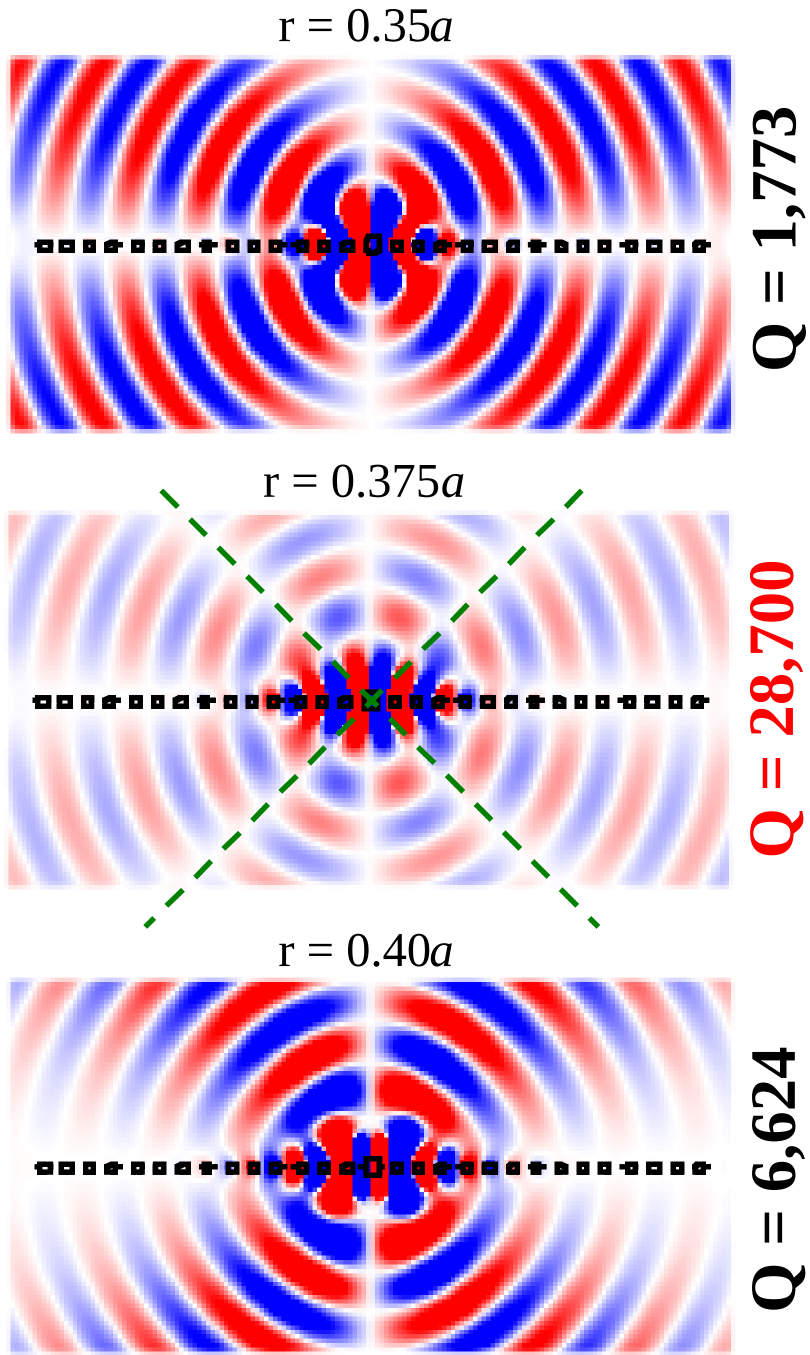
peak Q (cancellation) = transition to higher-order radiation

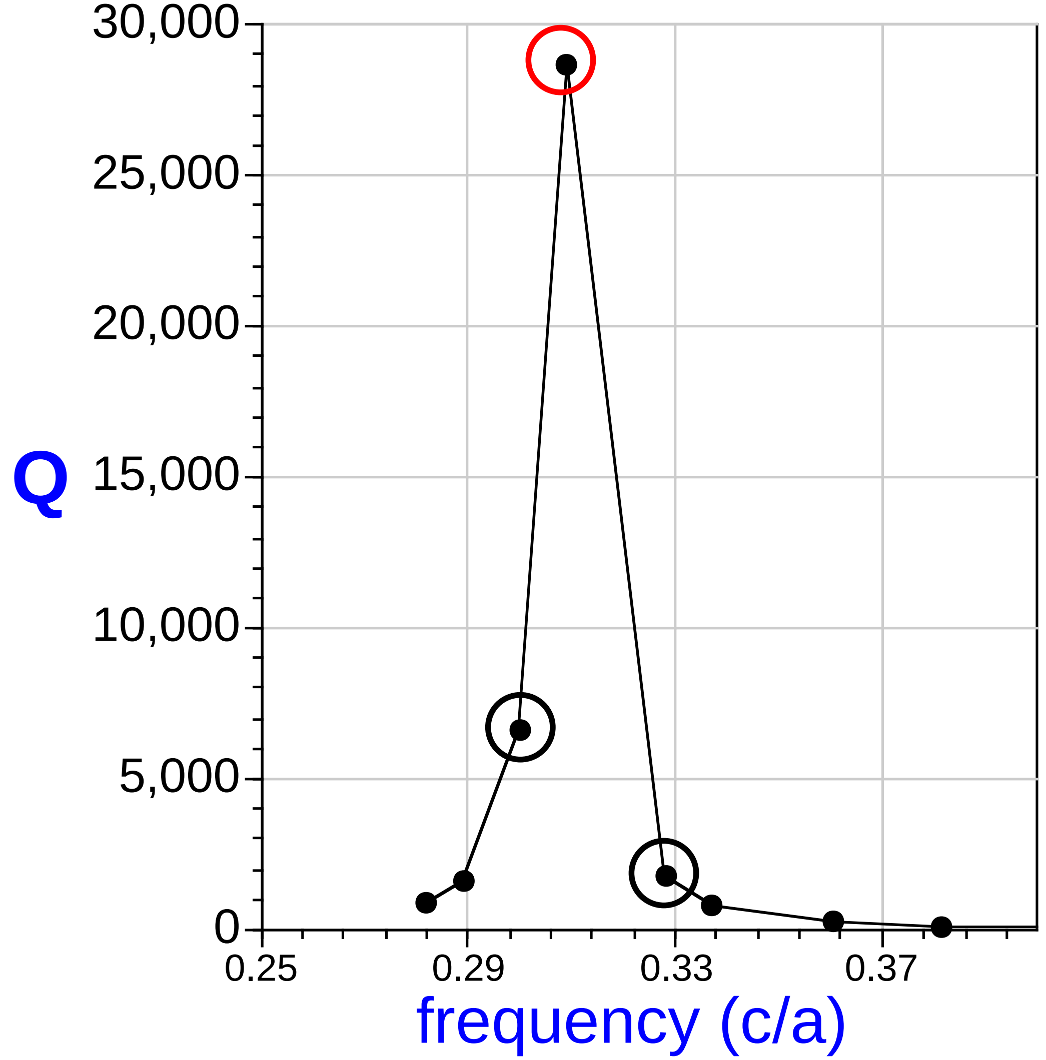
Multipoles in a 2d example



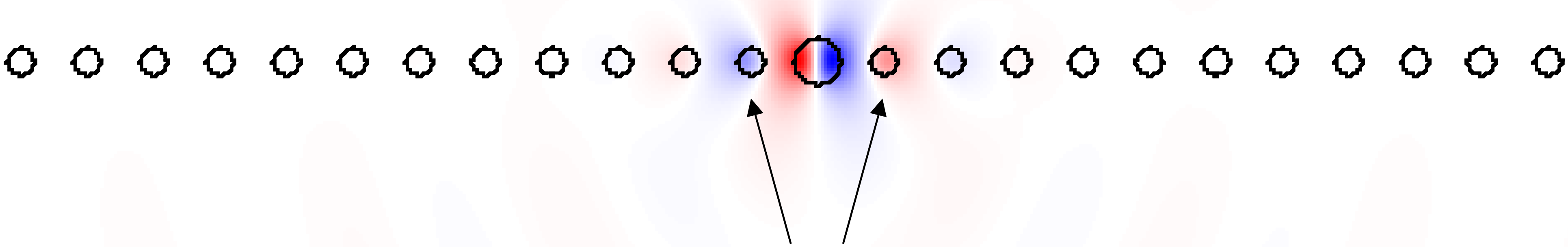
as we change the radius, ω sweeps across the gap

2d multipole cancellation





cancel a dipole by opposite dipoles…

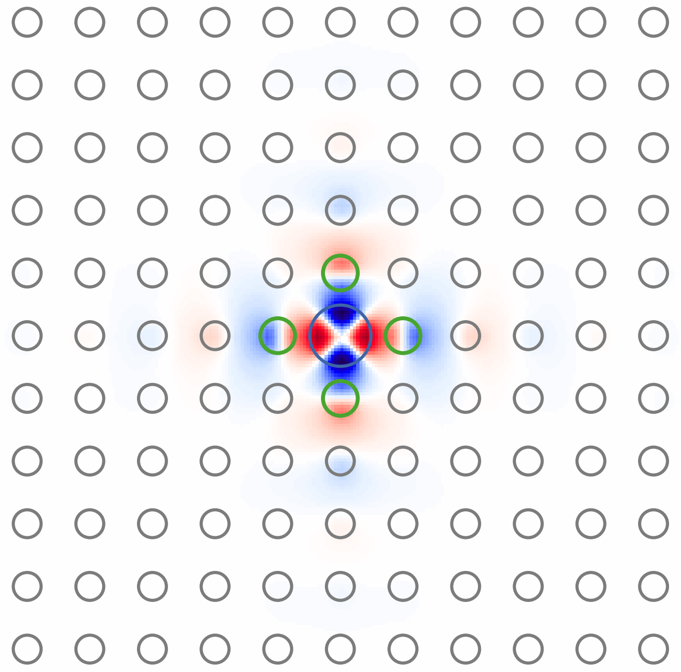


cancellation comes from
opposite-sign fields in adjacent rods

… changing radius changed balance of dipoles

3d multipole cancellation?

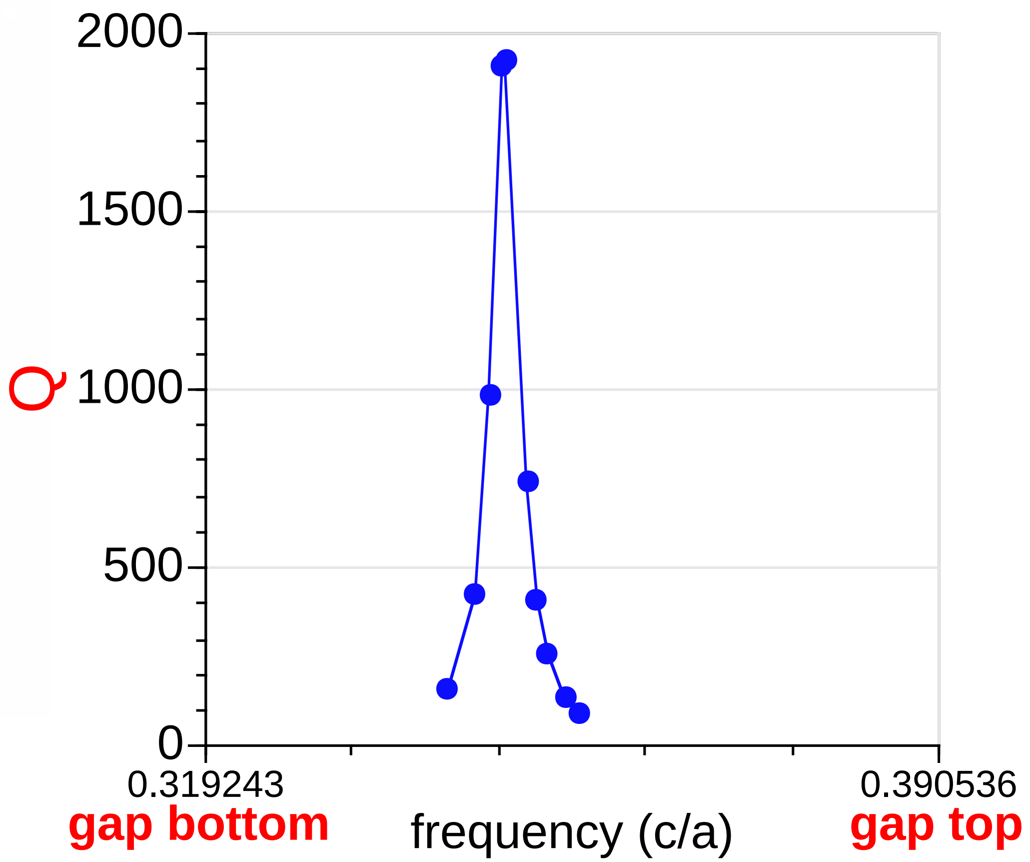
quadrupole mode



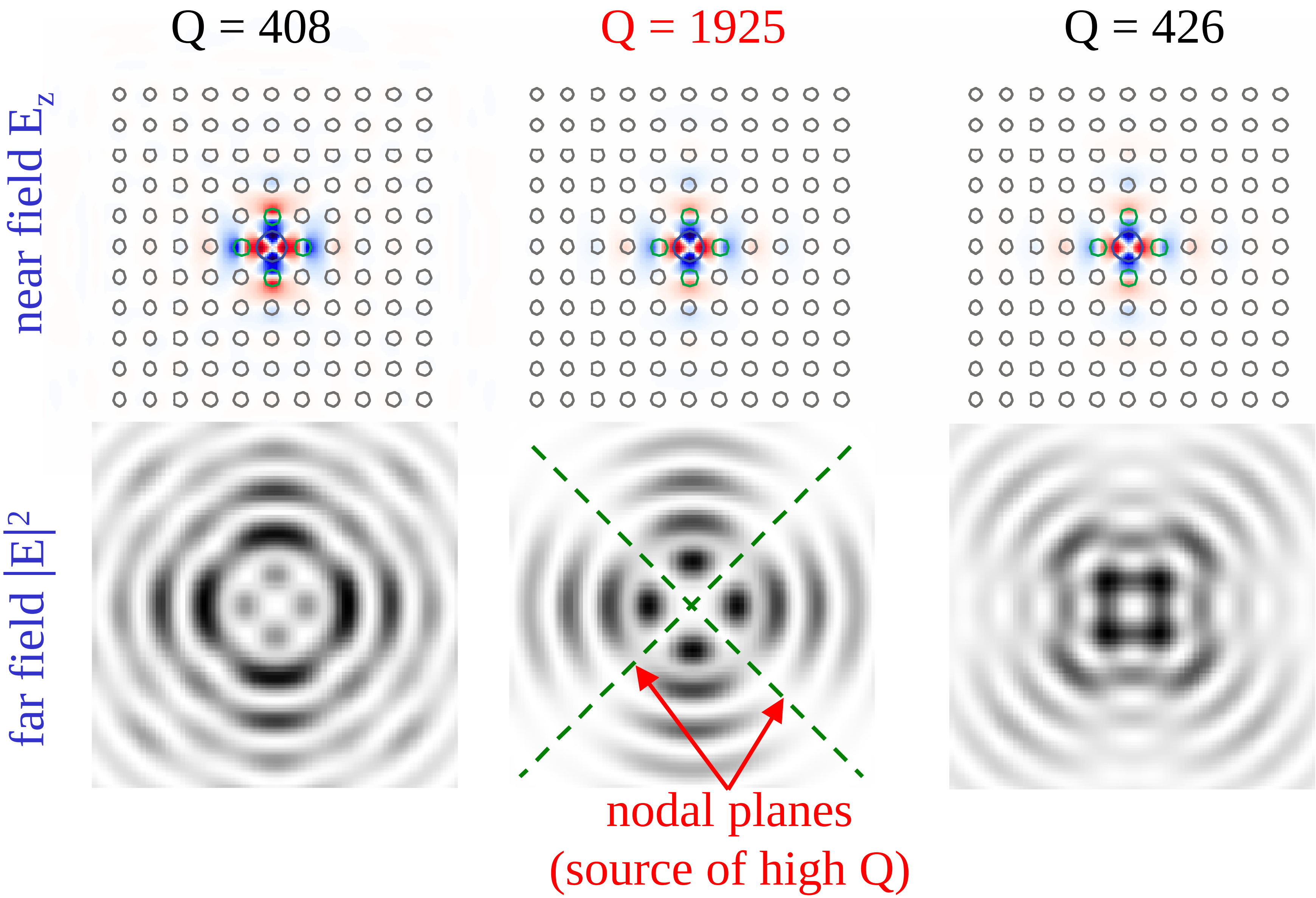
(E_z cross section)

enlarge center & adjacent rods

vary side-rod ε slightly
for continuous tuning
(balance central moment with opposite-sign side rods)



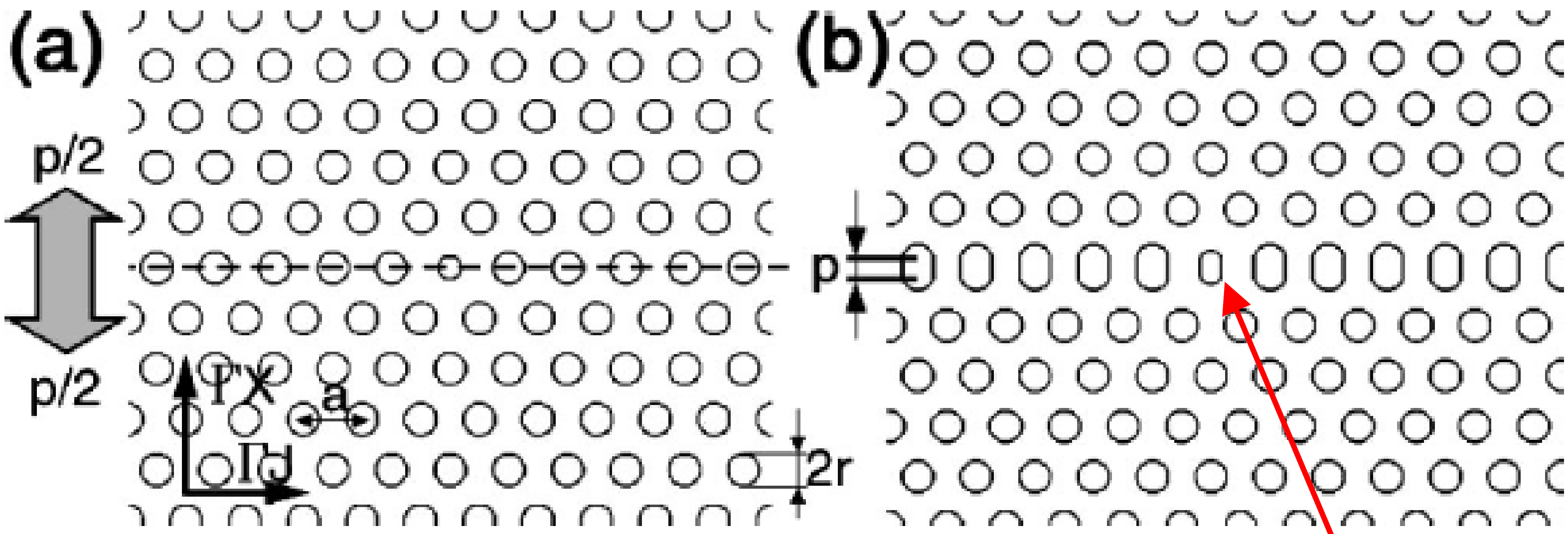
3d multipole cancellation



An Experimental (Laser) Cavity

[M. Loncar *et al.*, *Appl. Phys. Lett.* **81**, 2680 (2002)]

elongate row of holes



Elongation *p* is a tuning parameter for the cavity…

…in simulations, Q peaks sharply to ~10000 for $p = 0.1a$

(likely to be a multipole-cancellation effect)

* actually, there are two cavity modes; *p* breaks degeneracy

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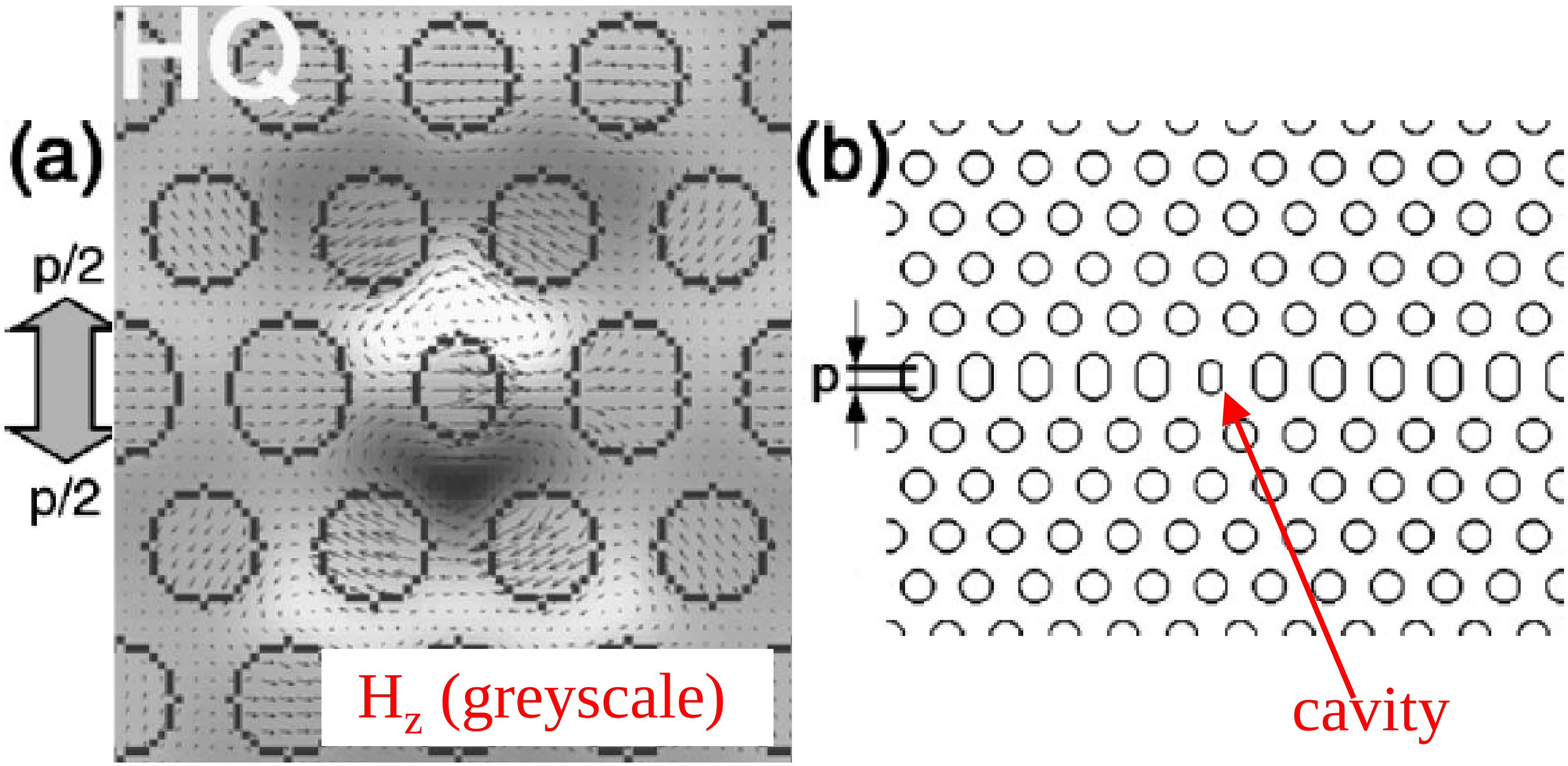

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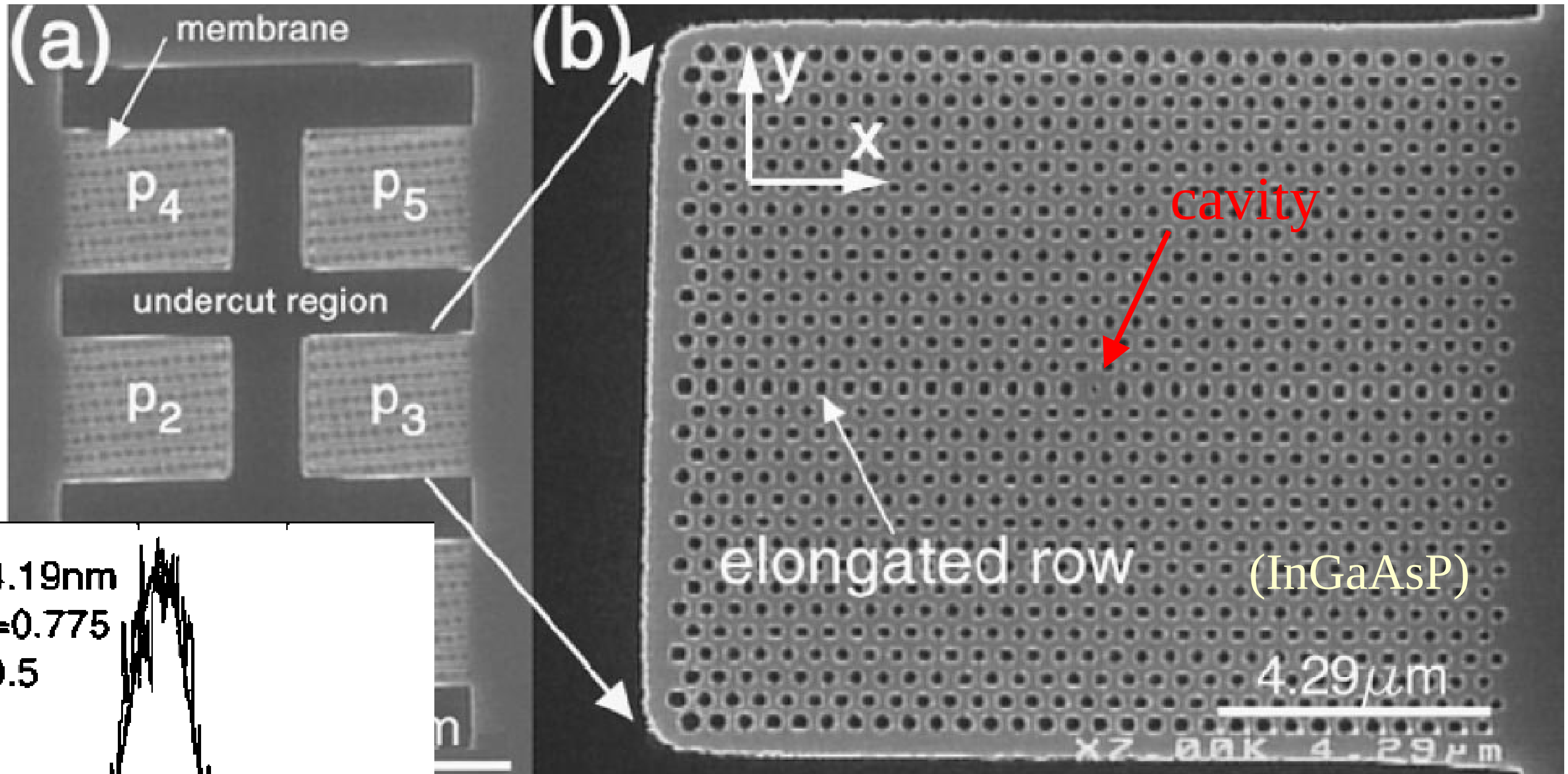
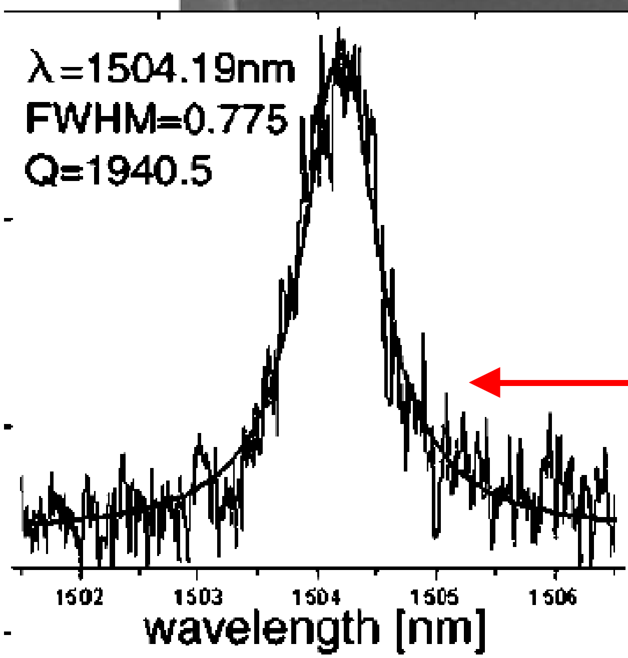




Q ~ 2000 observed from luminescence

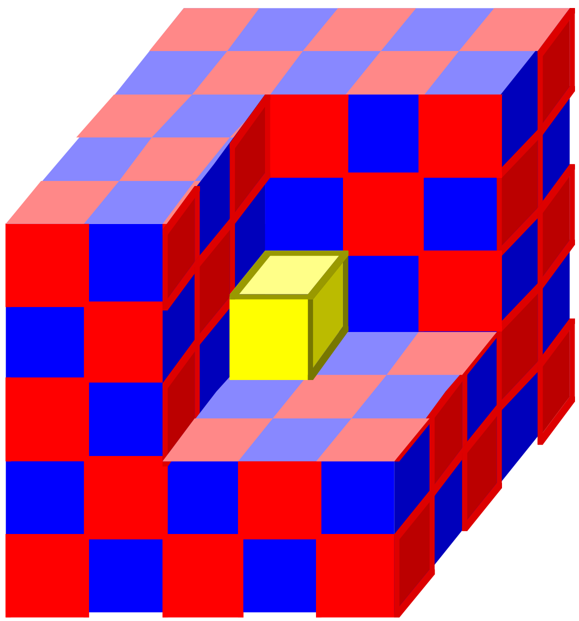
quantum-well lasing threshold of 214μW

(optically pumped @830nm, 1% duty cycle)

How can we get *arbitrary* Q with *finite* modal volume?

~~Only~~ one way:

a full 3d band gap



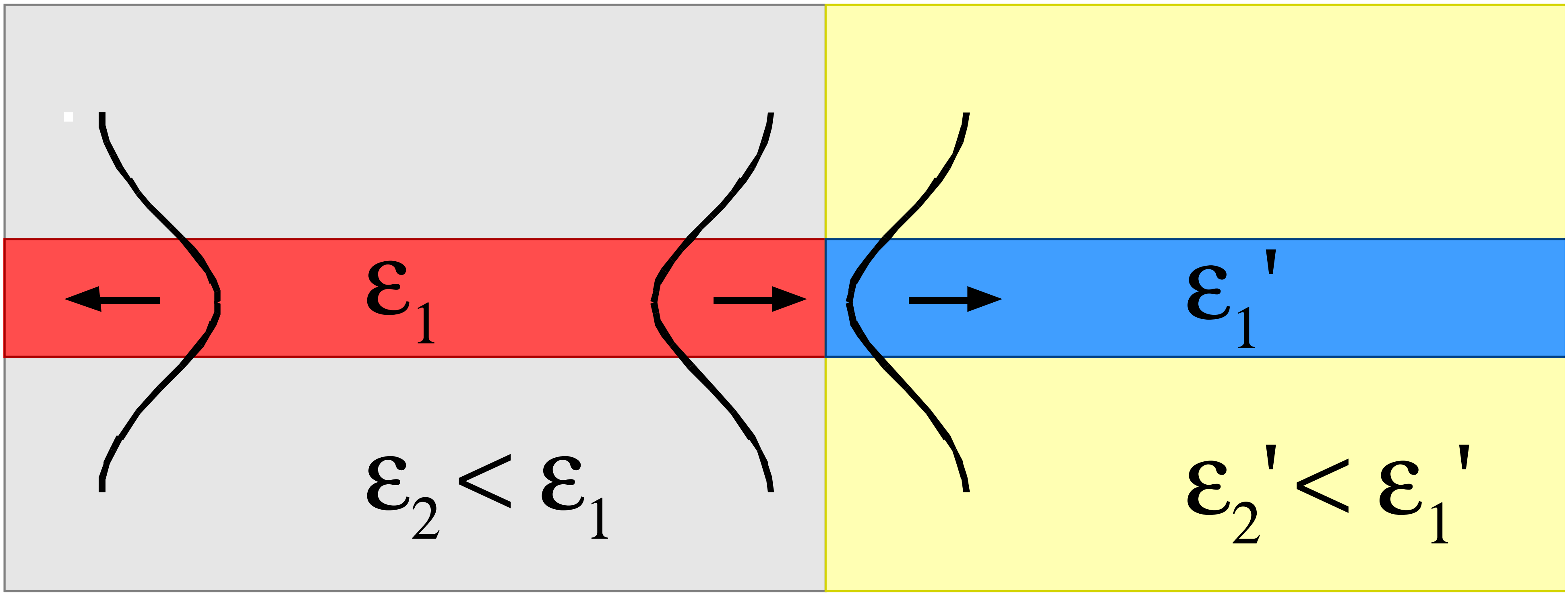
Now there are two ways.

[M. R. Watts *et al*., *Opt. Lett*. **27**, 1785 (2002)]

The Basic Idea, in 2d

start with:

junction of two waveguides



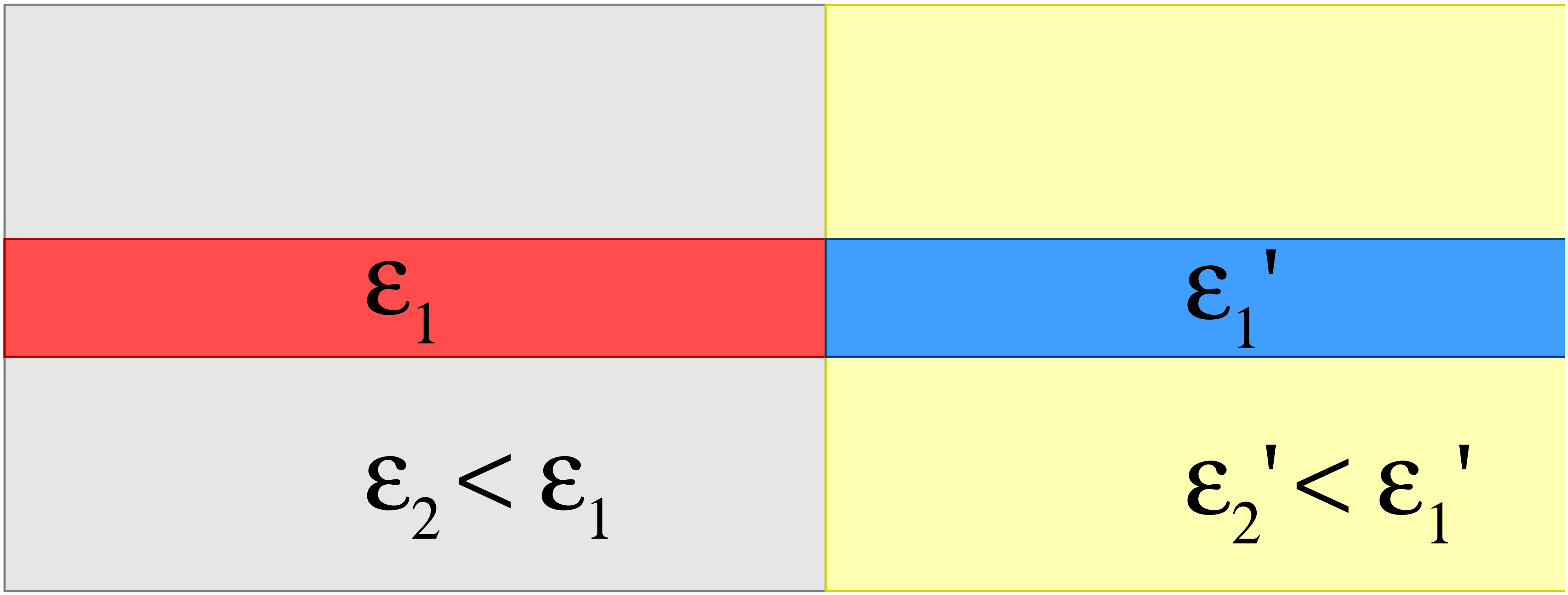
No radiation at junction

if the modes are perfectly matched

Perfect Mode Matching

requires:

same differential equations and boundary conditions



Match differential equations…

$$\varepsilon_2 - \varepsilon_1 = \varepsilon_2' - \varepsilon_1'$$

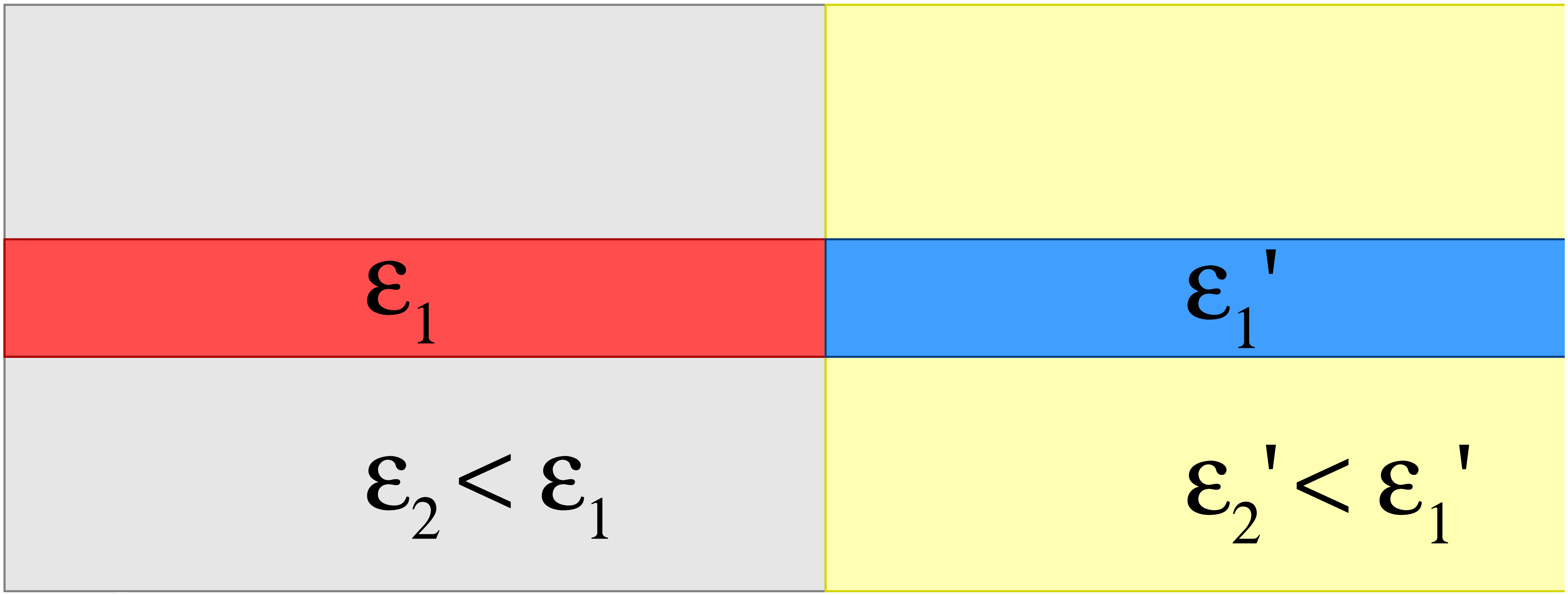
…closely related to separability

[S. Kawakami, *J. Lightwave Tech.* **20**, 1644 (2002)]

Perfect Mode Matching

requires:

same differential equations and boundary conditions



Match boundary conditions:

field must be TE

(note switch in TE/TM convention)

(*E* out of plane, in 2d)

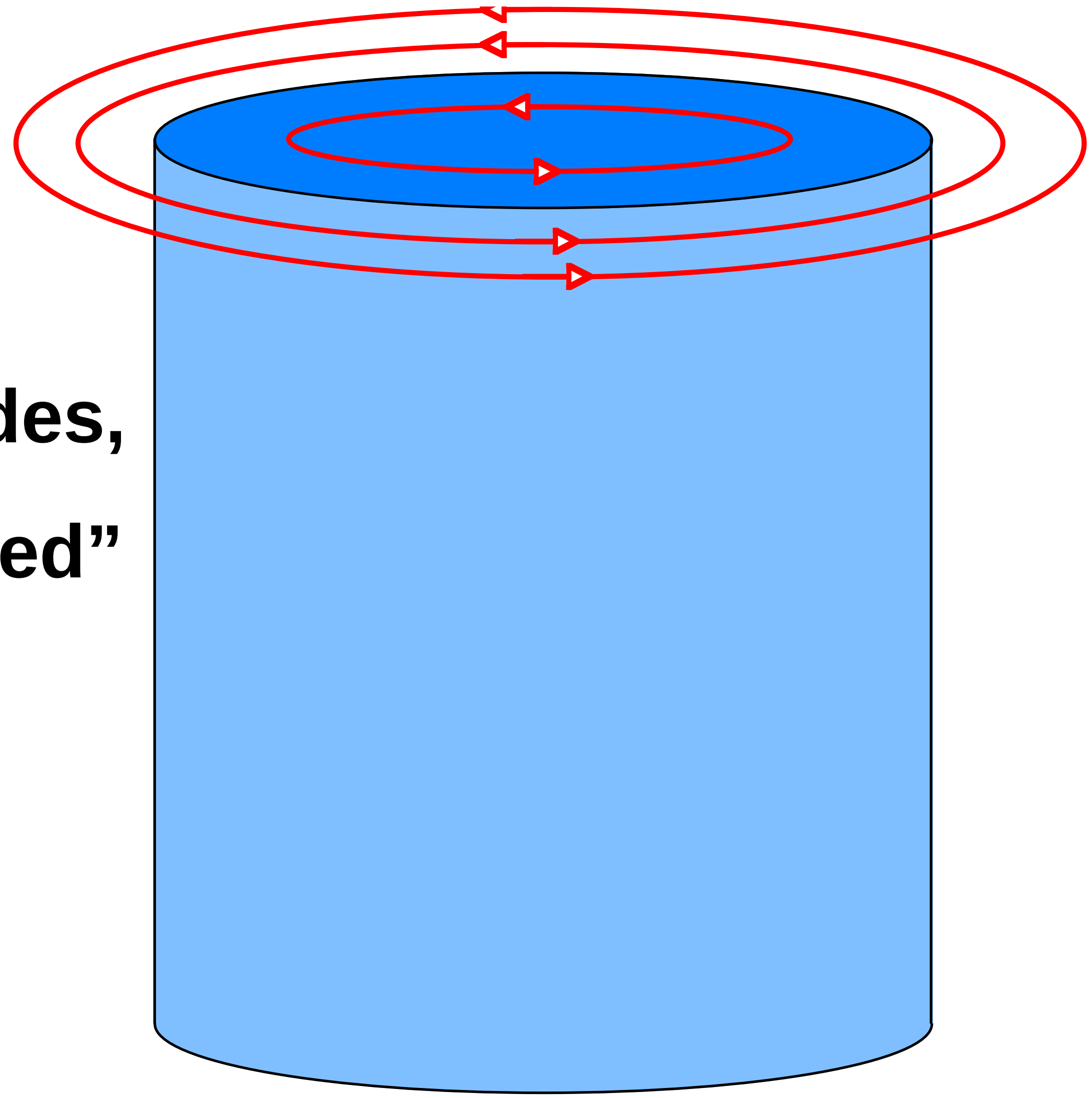
TE modes in 3d

for

cylindrical waveguides,

“azimuthally polarized”

TE_{0n} modes



A Perfect Cavity in 3d

(~ VCSEL + perfect lateral confinement)

Perfect index confinement

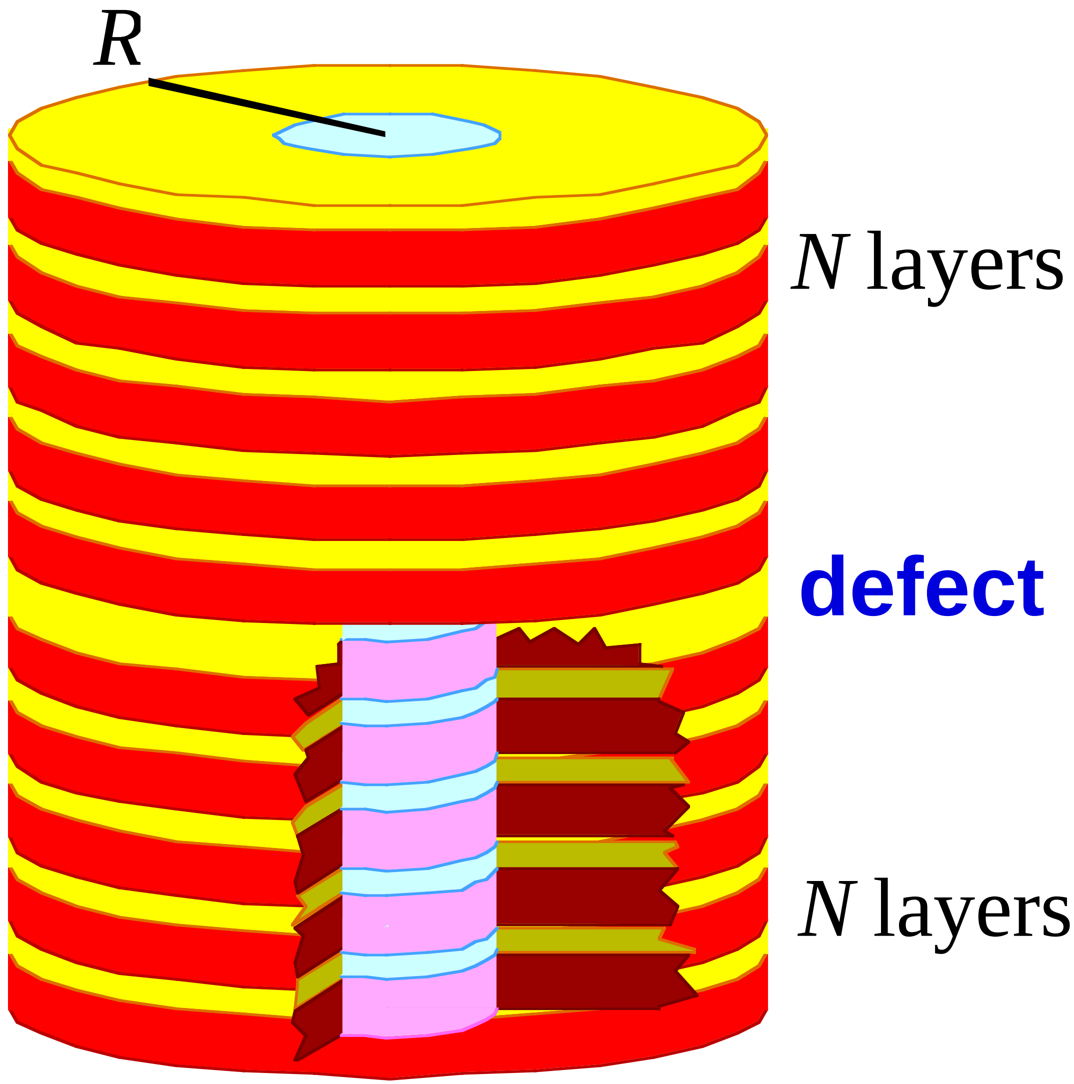
(no scattering)

+

1d band gap

=

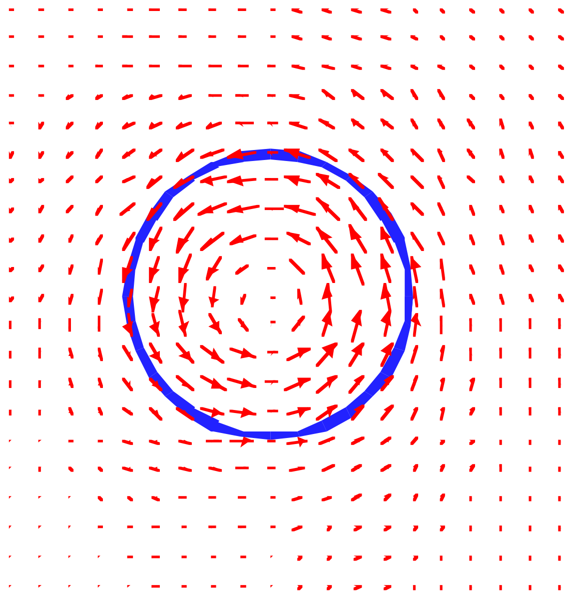
3d confinement

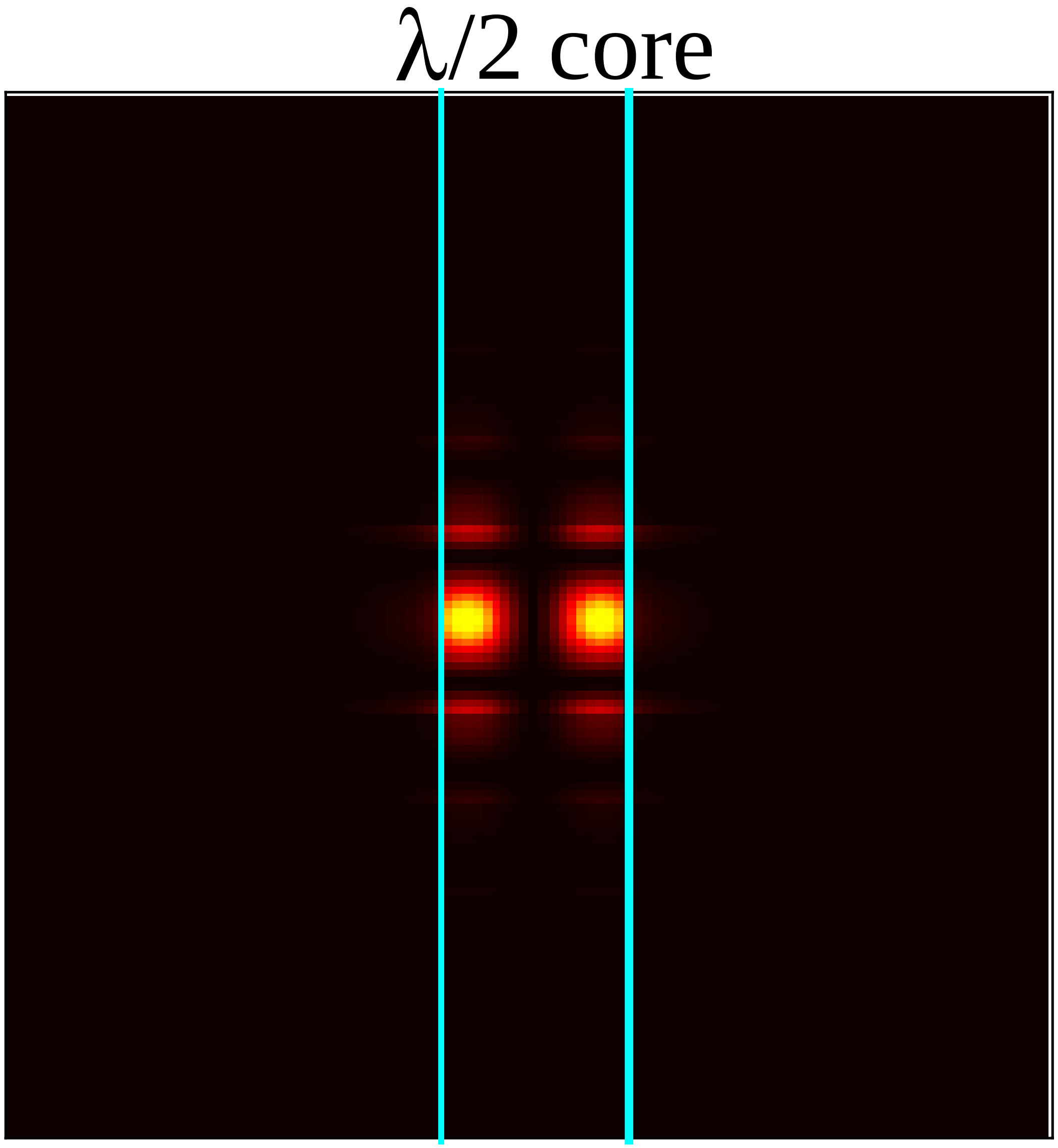


A Perfectly Confined Mode

$$\varepsilon_1, \varepsilon_2 = 9, 6$$

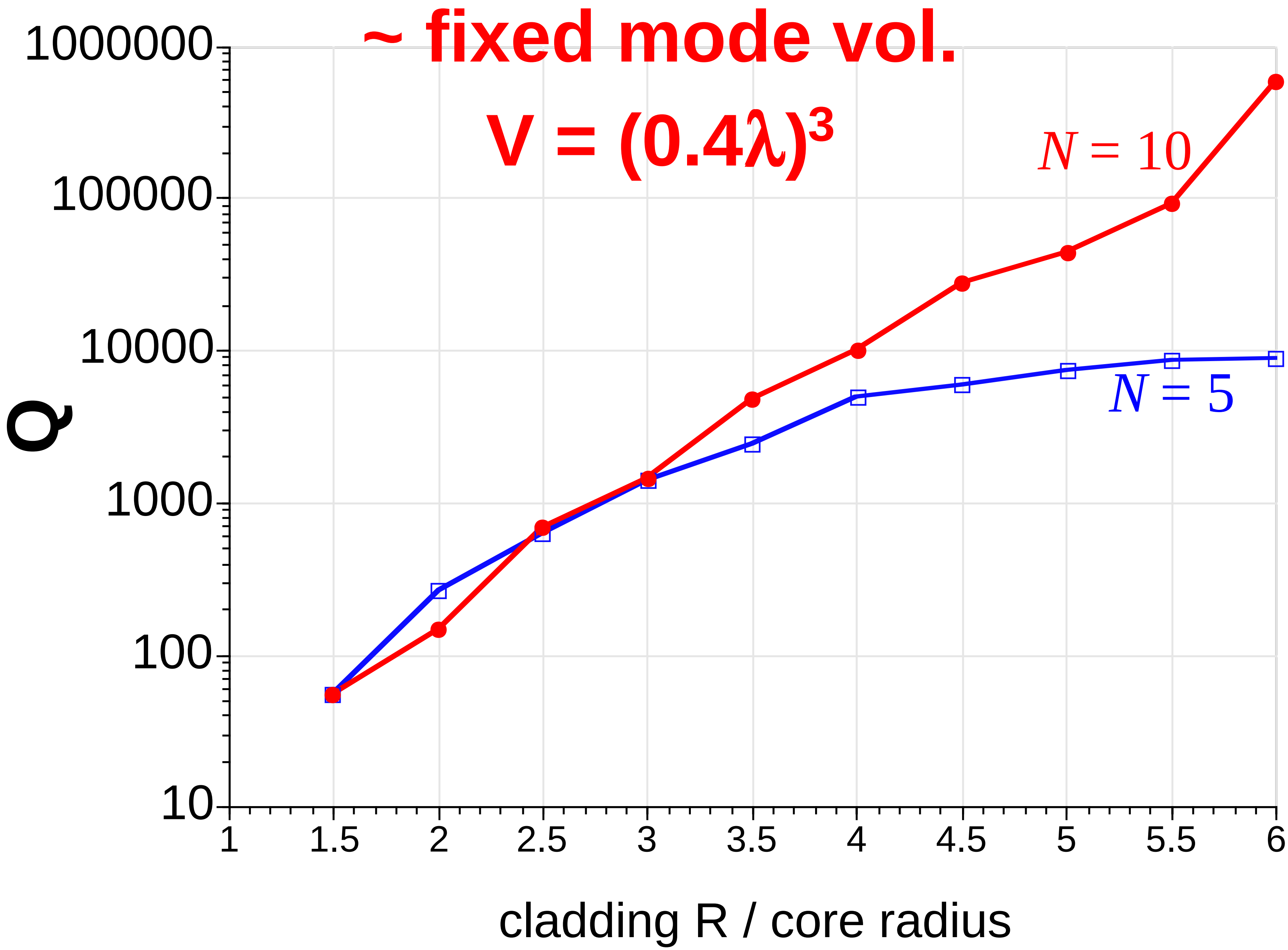
$$\varepsilon_1', \varepsilon_2' = 4, 1$$





E energy density, vertical slice

Q limited only by finite size



Q-tips

Three independent mechanisms for high Q:

Delocalization: trade off modal size for Q

Q_r grows monotonically towards band edge

Multipole Cancellation: force higher-order far-field pattern

*Q_r peaks **inside** gap*

New nodal planes appear in far-field pattern at peak

Mode Matching: allows arbitrary Q, finite V

Requires special symmetry & materials

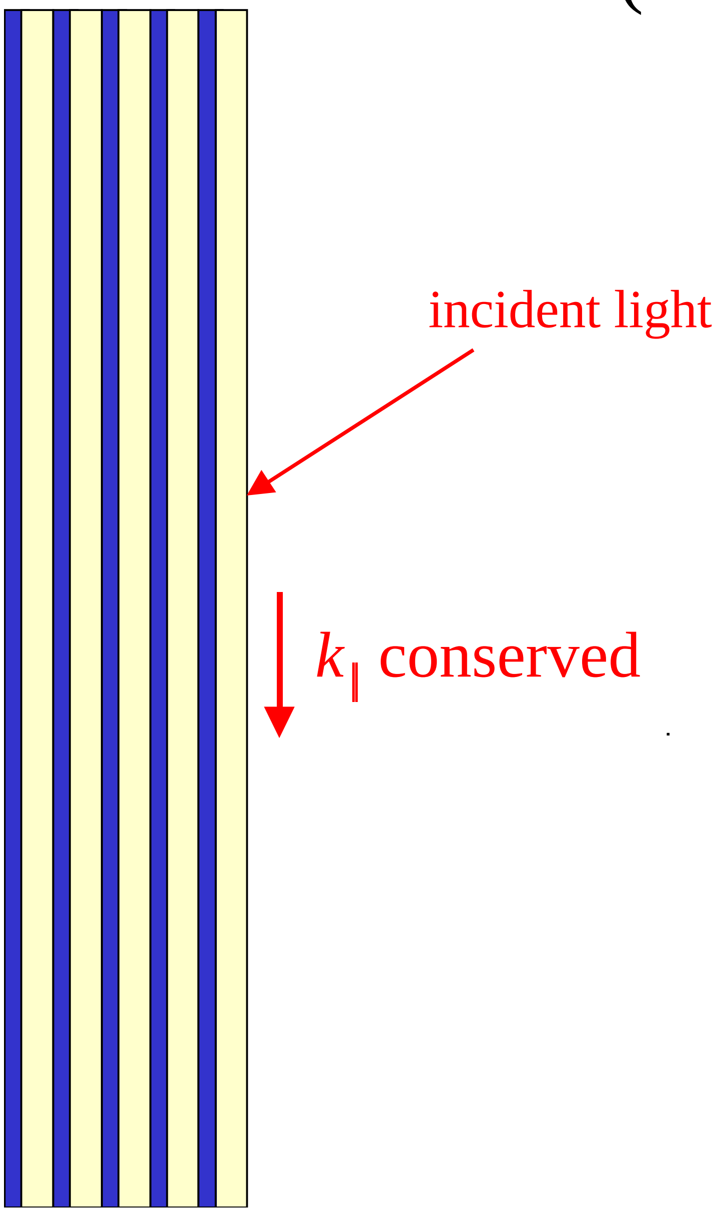
Forget these devices…

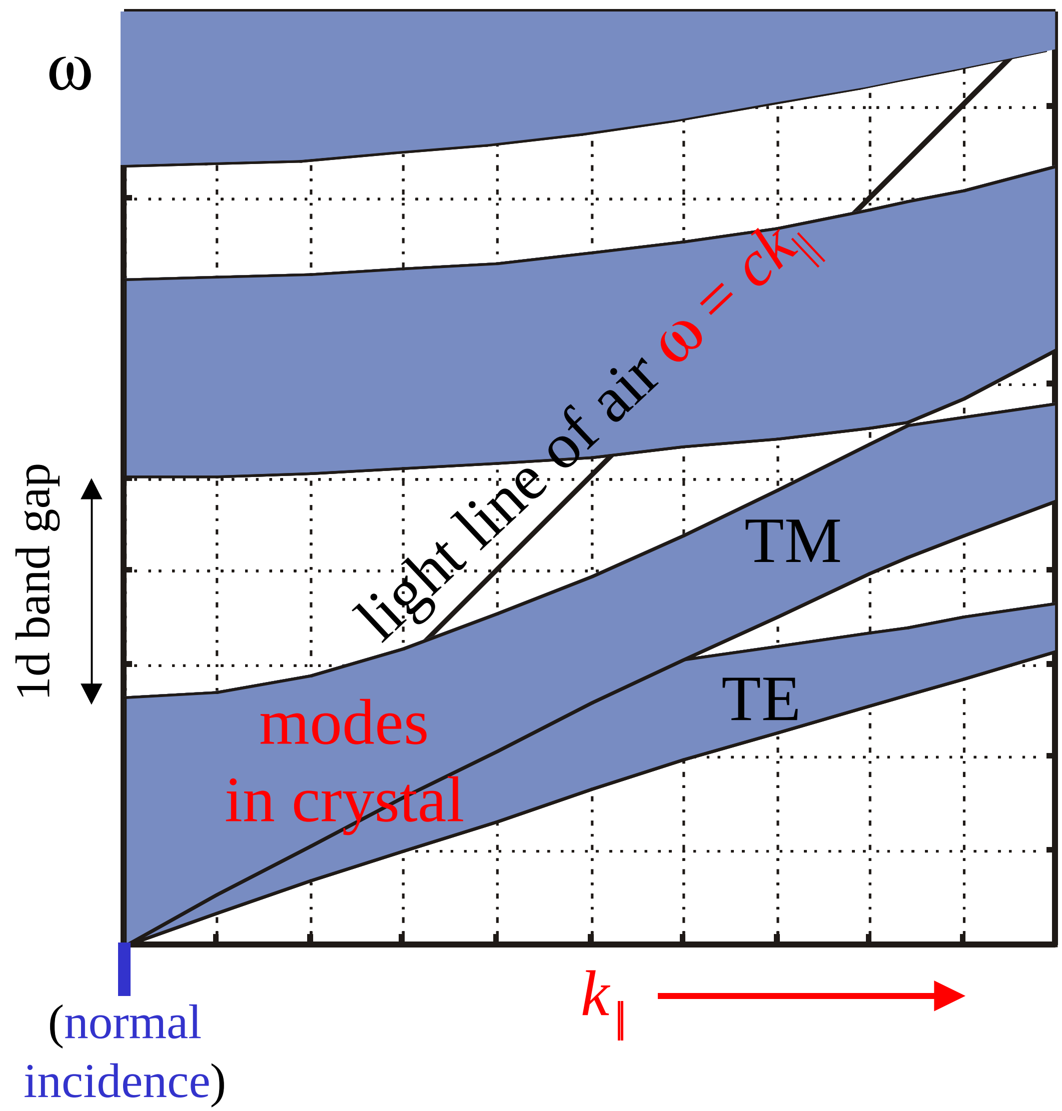
I just want a mirror.

ok

Projected Bands of a 1d Crystal

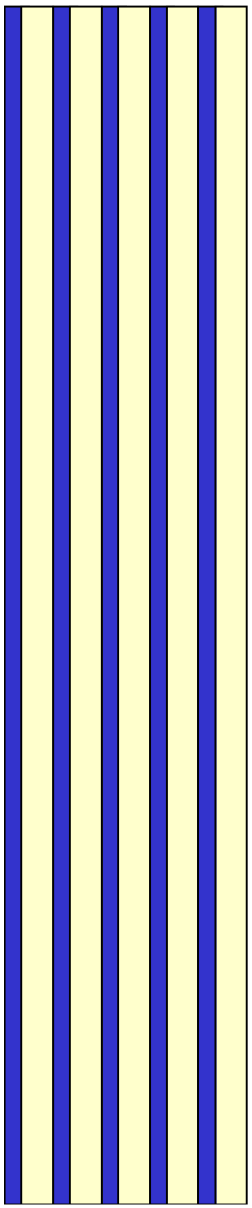
(a.k.a. a Bragg mirror)



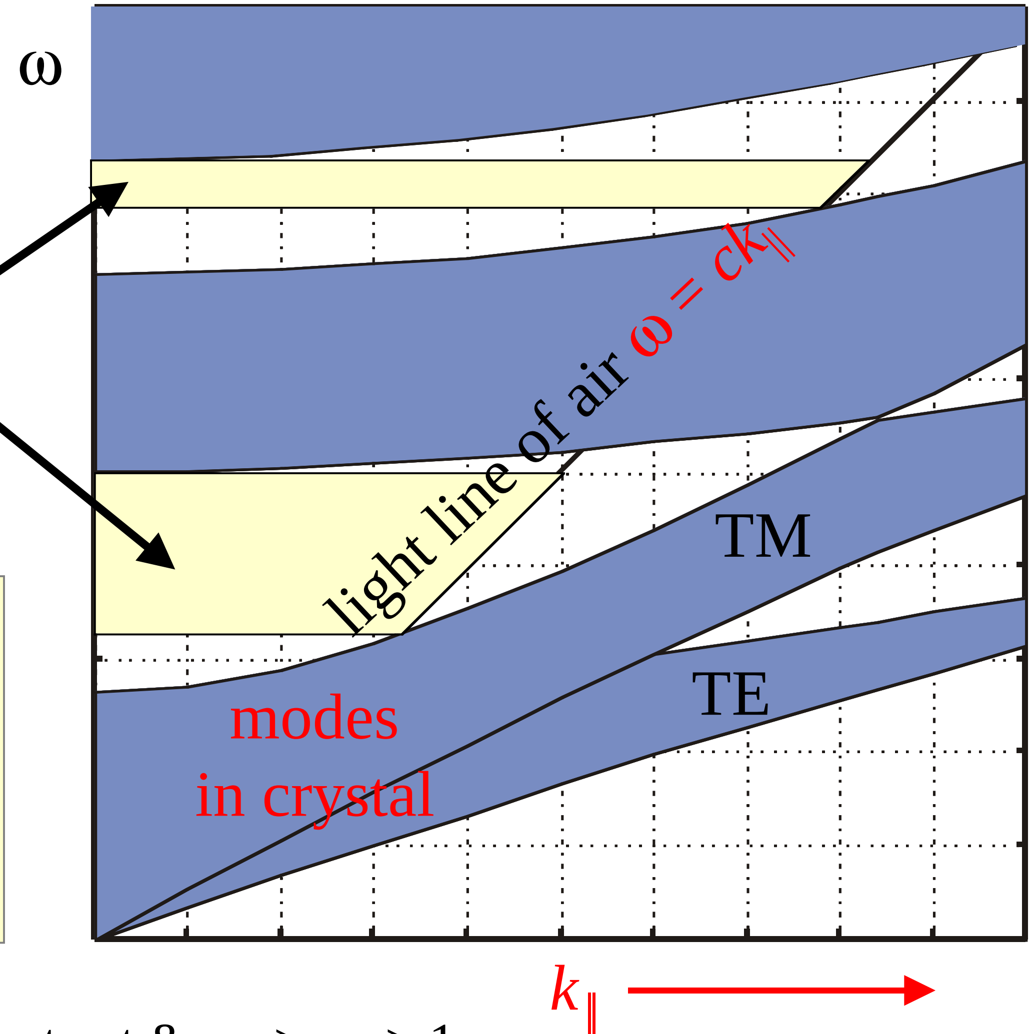


Omnidirectional Reflection

[J. N. Winn *et al*, *Opt. Lett.* **23**, 1573 (1998)]



in these ω ranges,
there is
no overlap
between modes
of air & crystal



all incident light
(any angle, polarization)
is reflected
from flat surface

needs: sufficient index contrast & $n_{hi} > n_{lo} > 1$

Omnidirectional Mirrors in Practice

[Y. Fink *et al*, *Science* **282**, 1679 (1998)]

contours of omnidirectional gap size