Single-photon all-optical switching using waveguide-cavity QED

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Abstract

This manuscript demonstrates single-photon switching using a single gating photon that interacts with materials exhibiting electromagnetically-induced transparency (EIT), embedded in photonic crystals (PhCs). An analytical model based on waveguide-cavity QED is constructed for our system, which consists of a PhC waveguide and a PhC microcavity containing a four-level EIT atom. It is solved exactly and analyzed using experimentally accessible parameters. It is found that the very strong coupling regime is required for lossless two photon quantum entanglement.
Several emerging technologies, such as integrated all-optical signal processing and all-optical quantum information processing, require strong and rapid interactions between two distinct optical signals [1]. Achieving this goal is a fundamental challenge because it requires a unique combination of large nonlinearities and low losses. The weak nonlinearities found in conventional media mean that large powers are required for switching. However, nonlinearities up to 12 orders of magnitude larger than those observed in common materials [2] with low losses can be achieved using EIT materials [2–4]. This has motivated several optical switching schemes involving a very small number of photons. Refs. [5–7] predict that EIT can create a photon blockade effect that allows the state of a cavity to be switched by a single photon. Ref. [8] predicts that ensembles of EIT atoms can be modulated to create quantum entangled states for a small number of photons. An alternative method is discussed in Ref. [9], whereby a laser beam can control the relative populations of a two-state system embedded in a PhC, which switches its transmission properties at low power levels.

To achieve switching, we utilize photonic crystals, which combine two intriguing capabilities: (1) guidance of light over long distances with low losses and (2) confinement of light to high quality factor microcavities with low modal volumes, which facilitate strong coupling between light and matter. Our scheme involves having one or a few 4-level EIT atoms strongly coupled to a PhC cavity, which in turn is coupled to a PhC waveguide. The presence (or absence) of photons at a gating frequency controls the transmission at a probe frequency. Ref. [10] has demonstrated this phenomenon for very low intensity fields using a semiclassical calculation. This manuscript extends that work by formulating a waveguide-cavity QED Hamiltonian for the same problem and solving it exactly, then demonstrating an approach to calculating these parameters from first principles. The emergence of new phenomena associated with the quantization of the probe and gate fields (e.g., Rabi-splitting) is discussed. Finally, it is shown that switching behavior can be achieved with single probe and gate photons, and the physical parameters needed to achieve such operations are calculated.

Consider the following design, illustrated in Fig. 1. There is a cavity which supports two resonant modes of frequencies $\omega_p$ and $\omega_{24}$, both coupled to a single four-level EIT atom with coupling strengths $g_{ij}$ and atomic transition frequencies $\omega_{ij}$, where $i$ and $j$ refer to the initial and final atomic states, respectively. The EIT dark state is created by adding a classical coupling field into the cavity with frequency $\omega_{23}$ and Rabi frequency $2\Omega_c$ – all other quantities are treated quantum mechanically. In general, any number of coupling schemes between the
cavity and one or more waveguides could be utilized. However, in this manuscript, the $\omega_p$ cavity mode is coupled to an adjacent single-mode waveguide with a radiative linewidth $\Gamma_p = \omega_p/2Q_p = V_p^2/v_g$, where $Q_p$ is the quality factor of the cavity mode, $V_p$ is the coupling strength and $v_g$ is the group velocity in the waveguide – its dispersion relation $\omega(k)$ is assumed to be approximately linear near the $\omega_p$ resonance. In this approximation, radiative couplings out of the system are neglected. Also, the $\omega_{24}$ resonance is assumed to have a much smaller decay rate $\Gamma_{24} = \omega_{24}/2Q_{24}$; therefore its decay is neglected (i.e., $Q_{24} \gg Q_p$).

In the absence of an atom, this design produces a Fano resonance: a Lorentzian lineshape for the reflection, centered around $\omega_p$ [11]. A photonic crystal implementation of this is shown in the upper left-hand corner of Fig. 1 – a triangular lattice of air holes in silicon with radius $0.48a$ that has a complete 2D photonic bandgap. A similar geometry has been used for quantum dots in photonic crystal microcavities, as in Ref. [12]. That experimental system exhibits a critical photon number $m_0 = \Gamma_3^2/2g^2 = 0.55$ and critical atom number $N_0 = 2\Gamma_p\Gamma_3/g^2 = 4.2$. Ideally both of these numbers would be less than one for quantum information processing [13]; it should be possible to achieve this goal with improvements in $Q$ or modal volume $V_{\text{mode}}$, or by placing several atomic or quantum dot systems in the same microcavity. Note that it could also be possible to achieve similar behavior with other physical systems, such as high-finesse Fabry-Perot optical microcavities [14], or ultrahigh-Q toroidal microresonators [15].
The Hamiltonian for this system will be given by:

\[
H/\hbar = \sum_k \omega_k a_k^\dagger a_k + \omega_p a^\dagger a + \sum_k V_p (a_k^\dagger + a_k)(a^\dagger + a) + \omega_{21}\sigma_{22} \\
+ (\omega_p - i\Gamma_3)\sigma_{33} + (\omega_p + \Delta\tilde{\omega}_{24})\sigma_{44} + \Omega_c (\sigma_{32} + \sigma_{23}) \cos (\omega_{23} t) \\
+ g_{13}(a^\dagger \sigma_{13} + a\sigma_{31}) + g_{24}(b^\dagger \sigma_{24} + b \sigma_{42})
\]

where \(a_k\) are the annihilation operators for waveguide states of wavevector \(k\) and frequency \(\omega_k\), \(a\) and \(b\) are the annihilation operators for cavity photon states of frequencies \(\omega_p\) and \(\omega_{24}\), respectively (which are considered in this paper to be singly occupied), \(\sigma_{ij}\) are the projection operators that take the atomic state from \(j\) to \(i\), \(\Gamma_3\) is the nonradiative decay rate of the third level, and \(\Delta\tilde{\omega}_{24} = \omega_{24} - \omega_p - i\Gamma_4\) is the complex detuning of the 2→4 transition from \(\omega_p\). Although \(\Delta\tilde{\omega}_{24}\) is predominantly real, in general there is an imaginary part \(-i\Gamma_4\), corresponding to absorption losses in the fourth level. However when the detuning greatly exceeds the decay rate of the upper level, this contribution may be neglected. Losses from the second atomic level are also neglected, since typically it is a metastable state close to the first atomic level ground state in energy. Finally, the first and third atomic levels are coupled with strength \(g_{13}\) to a cavity photon of frequency \(\omega_p\) and the second and fourth atomic levels are coupled with strength \(g_{24}\) to a cavity photon of frequency \(\omega_{24}\).

Following Refs. [6] and [11], the Hamiltonian in equation (1) can be rewritten in real space and separated into a diagonal part:

\[
H_o/\hbar = \omega_p \int dx \left[ a_R^\dagger(x) a_R(x) + a_L^\dagger(x) a_L(x) \right] \\
+ \omega_p(a^\dagger a + b^\dagger b + \sigma_{33} + \sigma_{44}) + \omega_{21}\sigma_{22},
\]

where \(a_L\) and \(a_R\) refer to left and right moving waveguide photons, respectively, as well as an interaction part:

\[
H_I/\hbar = \int dx \left[ a_R^\dagger(x)(-iv_y \partial_x - \omega_p)a_R(x) + a_L^\dagger(x)(iv_y \partial_x - \omega_p)a_L(x) \\
+ V_p \delta(x)(a_R^\dagger(x)a + a_R(x)a^\dagger + a_L^\dagger(x)a + a_L(x)a^\dagger) + \Omega_c (\sigma_{23} + \sigma_{32}) \\
+ g_{13}(a^\dagger \sigma_{13} + a\sigma_{31}) - i\Gamma_3\sigma_{33} + \Delta\tilde{\omega}_{24}\sigma_{44} + g_{24}(b\sigma_{42} + b^\dagger \sigma_{24}) \right]
\]

via the interaction picture, where the total system Hamiltonian is given by \(H = H_o + H_I\).
The ground state for the system can be written as:

\[
|\psi_k\rangle = \left\{ \int dx \left[ \phi_{k,R}^+ (x) a_R^+ (x) + \phi_{k,L}^+ (x) a_L^+ (x) \right] + e_k a^+ + f_k \sigma_{31} + h_k \sigma_{21} + p_k \sigma_{41} b \right\} |0,0,1\rangle_{phc} \otimes |1\rangle_{atom}
\]

where:

\[
\phi_{k,R}^+ (x) = e^{ikx} [\theta (-x) + i \theta (x)]
\]

\[
\phi_{k,L}^+ (x) = re^{-ikx} \theta (-x),
\]

\(e_k\) is the probability amplitude of the cavity photon at \(\omega_p\), and \(f_k, h_k,\) and \(p_k\) are the occupations of the 3rd, 2nd, and 4th atomic levels, respectively. \(t\) and \(r\) are the waveguide transmission and reflection amplitudes, respectively. All of these parameters are determined when the eigenequation is solved below. \(|0,0,1\rangle_{phc} \otimes |1\rangle_{atom}\) corresponds to a “ground state” consisting of a direct product of a photonic state (\(phc\)) and an atomic state (\(atom\)). The photonic state consists of zero photons in the waveguide, zero photons in the cavity at \(\omega_p\), and one photon in the cavity at \(\omega_{24}\), respectively. The atomic state consists of a single atom in its ground state. In this model, the number of \(\omega_{24}\) photons is restricted to be zero or one.

In the case of \(n\ \omega_{24}\) photons, the coupling constant \(g_{24} \rightarrow g'_{24} = g_{24} \sqrt{n}\).

Applying the Hamiltonian, equation (3), to the time-independent eigenvalue equation

\[ H_I |\psi_k\rangle = \hbar \epsilon_k |\psi_k\rangle, \]

where \(\epsilon_k = \omega - \omega_p\), and solving for the reflection coefficient yields

\[ |r(\epsilon_k)|^2 = |\Gamma_p / (\xi - i \Gamma_p)|^2, \]

where:

\[
\xi = \epsilon_k - \frac{g_{13}^2}{\epsilon_k + i \Gamma_3 - \frac{\Omega^2}{\epsilon_k - g_{24}^2/\epsilon_v - \Delta_{\omega_{24}}}}
\]

The parameters \(g_{13}, V_p\) (or \(\Gamma_p\)), and \(v_g\) of equation (3) can be determined from a numerical solution to Maxwell’s equations (e.g., via [16]) as follows. First, the cavity mode is excited by a source, and the modal volume of the cavity is found from the field patterns by \(V_{mode} = (\int_{mode} d^3 x |E|^2) / e |E_{max}|^2\). One can then apply the formula \(g_{13} = \sqrt{\pi \epsilon^2 f_{13}/meV_{mode}} \) [17], where \(\epsilon\) is the elementary electric charge, \(\epsilon\) is the dielectric constant of the medium in which the atomic system is embedded, \(m\) is the free electron mass, and \(f_{13}\) is the oscillator strength for the \(|1\rangle \rightarrow |3\rangle\) transition (1/2 in Na [2]). The linewidth \(\Gamma_p\) can be calculated by examining the decay rate of the field in the cavity mode. Finally, the waveguide group velocity is given by \(v_g = \frac{d\omega(k)}{dk} \bigg|_{\omega = \omega_p}\).
First, consider the case of a 2-level atomic system (i.e., $\Omega_c = 0$, $g_{24} = 0$), with a waveguide coupling $\Gamma_p$ and a non-radiative decay rate $\Gamma_3$. For a fixed atom-photon coupling $g_{13}$ and zero non-radiative absorption, the single resonant mode at $\epsilon_k = 0$ experiences a Rabi splitting into two orthogonal linear superpositions of the cavity and atom modes at $\epsilon_k = \pm g_{13}$. As long as one remains in the strong coupling regime $g_{13} > \Gamma_3/2$, the absorption for all frequencies increases nearly linearly with $\Gamma_3$.

However, in the opposite regime of weak coupling ($g_{13} < \Gamma_3/2$), the normal modes of the system are mostly photonic (lossless) or mostly atomic (very lossy). This phenomenon eliminates the Rabi splitting and gives rise to a reflection nearly indistinguishable from a system without an atom for sufficiently large $\Gamma_3$.

Now, consider a 3-level atomic system without losses. Compared to the 2-level system, a third mode, corresponding to the dark state of the EIT atom, will emerge at $\epsilon_k = 0$ between the previously observed Rabi-split peaks. The eigenstate associated with the dark state is given by $|\psi\rangle_{\text{dark}} = \left[a^\dagger - (g_{13}/\Omega_c)\sigma_{21}\right]|0,0,0\rangle_{\text{phc}} \otimes |1\rangle_{\text{atom}}$. The width of the central peak is expected to scale as $(\Omega_c/g_{13})^2$ for small $\Omega_c/g_{13}$ [8]. If one substitutes the expression given in Ref. [17] for $g_{13}$, one obtains the classical results found in Refs. [2, 10]. Meanwhile, the width of the side peaks is set by $\Gamma_p$ and remains roughly constant as one tunes the parameters of the system.

In Fig. 2, $g_{13}/\Omega_c = 2$ while $g_{13}$ is varied. It is shown that as $g_{13}$ is decreased, the central resonance width stays constant, while the distance between the central and Rabi-split peaks becomes smaller. For use in applications, it therefore seems optimal to have a large Rabi splitting, corresponding to the very strong coupling limit, which can also be viewed as corresponding to critical photon and atom numbers much less than one. The experimental values for a system with a single quantum dot emitting a single photon observed in Ref. [12] correspond to a regime where $g_{13} \approx \Gamma_p$ – specifically, they find that for operation at $\lambda = 1.182 \mu\text{m}$, $g_{13} = 20.5 \text{ GHz}$ and $\Gamma_p = 21.5 \text{ GHz}$; note that PhC microcavities are optimal for simultaneously decreasing $\Gamma_p$ and increasing $g_{13}$.

Now, consider a 4-level system with an $\omega_{24}$ photon present. Two possible effects can be induced by the $\omega_{24}$ photon. When $\omega_{24}$ is close to the cavity photon frequency, an Autler-Townes doublet is observed; when detuned, an AC-Stark shift will be induced in this system [6, 10]. The latter effect has been suggested as a switching mechanism in Refs. [10, 18, 19]. This can be shown by using equation (6) to calculate the poles of the EIT term in the reflection, i.e.,
FIG. 2: Waveguide reflection for a lossless 3-level EIT atom for the four labelled values of the atomic coupling strength $g_{13}$, in GHz. The radiation rate $\Gamma_p = 21.5$ GHz and the ratio $g_{13}/\Omega_c = 2$ are fixed. Larger $g_{13}$ produces larger peak separations (the blue curve shows Rabi peaks outside of the plot), favorable for switching.

Set $\epsilon_k = \frac{g_{24}^2}{(\epsilon_k - \Delta\tilde{\omega}_{24})} = 0$, which yields $\epsilon_k = \pm g_{24}$ for no detuning, and $\epsilon_k \approx -\frac{g_{24}^2}{\Delta\tilde{\omega}_{24}}$ for a large detuning, matching the semi-classical result found in Ref. [10].

Single-photon switching is obtained when the reflection peak is shifted by an amount greater than its width, via the presence or absence of one $\omega_{24}$ photon. In order to achieve this goal, one can take two different approaches. First, in the regime where $g_{13} \approx \Gamma_p$, as in Ref. [12], one can introduce an absorption via $\Gamma_3 \neq 0$, and thus absorb the majority of light not coupled to the dark state. In Fig. 3, the reflection and absorption are plotted for an optimized value of $\Gamma_3 = 30$ GHz, both before and after switching. As shown, reflections at the Rabi-split frequencies are decreased substantially (to about 40%), while full reflection is still observed at the central, EIT-narrowed peak. Furthermore, in the presence of a single detuned $\omega_{24}$ photon, it is possible to switch the peak reflection frequency by an amount greater than the EIT-narrowed central peak width. A second approach (if producing a large nonradiative decay $\Gamma_3$ is difficult in a single-atom device) is to enhance the ratio $g_{13}/\Gamma_p$ by either decreasing $\Gamma_p$ or $V_{\text{mode}}$, or by increasing the number of atoms from one to $N$. 
FIG. 3: Waveguide reflection (blue) and absorption (red) in the absence (solid) and presence (dashed) of an $\omega_{24}$ photon, demonstrating nonlinear single-photon switching ($\Gamma_p = 21.5$ GHz, $g_{13} = 20.5$ GHz, $\Omega_c = 2$ GHz, $\Gamma_3 = 30$ GHz, $g_{24} = 8$ GHz, and $\Delta\tilde{\omega}_{24} = 30$ GHz).

(which increases the coupling constant $g_{13} \rightarrow g'_{13} = g_{13}\sqrt{N}$ [20]); this final effect persists for more than one photon as long as the number of photons $n \ll N$ [20]. This collective Rabi oscillation separates the Rabi-split peaks much further from the central peak. Fig. 4 shows switching exploiting this phenomenon based on parameters from Ref. [12] and using $N = 25$ (which is equivalent to lowering $\Gamma_p$ or $\sqrt{V_{\text{mode}}}$ by a factor of 5). The advantage of this scheme is that, compared to the $\Gamma_3 \neq 0$ technique, one obtains a substantially greater tuning range and contrast (the difference between the peaks and the troughs). In conclusion, the reflection peak of a waveguide-cavity system can be switched in and out of resonance by a single gating photon, assuming realistic experimental parameters. Thus, one photon can be used to gate another photon of a different frequency. Under proper circumstances, this can give rise to two-photon entangled states. The integration of microcavities and waveguides in the same photonic crystal means that the entanglement could be preserved, in principle, throughout the system, which could be of use for quantum information processing [13].
FIG. 4: Waveguide reflection with (dashed) and without (solid) an $\omega_{24}$ photon, demonstrating switching, where multiple (25) EIT atoms have been used to push the Rabi-split peaks farther away in the presence of negligible loss ($\Gamma_p = 21.5 \text{ GHz}, g_{13} = 102.5, \Omega_c = 30 \text{ GHz}, \Gamma_3 = 0, g_{24} = 30 \text{ GHz and } \Delta \tilde{\omega}_{24} = 20 \text{ GHz}$).

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